SIGNALS AND SYSTEMS

1. Syllabus

Unit – I **SIGNAL ANALYSIS**

Introduction signals & Systems, Analogy between vectors and signals, Orthogonal signal space, Signal approximation using orthogonal functions, Mean square error , Closed or complete set of orthogonal functions, Orthogonality in complex functions, Exponential and sinusoidal signals, Impulse function, Unit step function, Signum function.

FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS

Representation of Fourier series, Continuous time periodic signals, properties of Fourier series, Dirichlet's conditions, Trigonometric Fourier series and Exponential Fourier series, Complex Fourier spectrum.

Unit – II **FOURIER TRANSFORMS & SAMPLING**

Deriving Fourier transform from Fourier series, Fourier transform of arbitrary signal, Fourier transform of standard signals, Fourier transform of periodic signals, Properties of Fourier transforms, Fourier transforms involving impulse function and Signum function, Introduction to Hilbert Transform. Sampling theorem –Graphical and analytical proof for Band Limited Signals, impulse sampling, Natural and Flat top Sampling, Reconstruction of signal from its samples, effect of under sampling – Aliasing, Introduction to Band Pass sampling

Unit – III **SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS**

Linear system, impulse response, Response of a linear system, Linear time invariant (LTI) system, Linear time variant (LTV) system, Transfer function of a LTI system, Filter characteristics of linear systems, Distortion less transmission through a system, Signal bandwidth, system bandwidth, Ideal LPF, HPF and BPF characteristics, Causality and Poly-Wiener criterion for physical realization, Relationship between bandwidth and rise time.

Unit – IV **CONVOLUTION AND CORRELATION OF SIGNALS**

Concept of convolution in time domain and frequency domain, Graphical representation of convolution, Convolution property of Fourier transforms, Cross correlation and auto correlation of functions, properties of correlation function, Energy density spectrum, Parseval's theorem, Power density spectrum, Relation between auto correlation function and energy/power spectral density function, Relation between convolution and correlation, Detection of periodic signals in the presence of noise by correlation, Extraction of signal from noise by filtering.

Unit – V **LAPLACE TRANSFORMS**

Review of Laplace transforms, Partial fraction expansion, Inverse Laplace transform, Concept of region of convergence (ROC) for Laplace transforms, constraints on ROC for various classes of signals, Properties of L.T's relation between L.T's, and F.T. of a signal, Laplace transform of certain signals using waveform synthesis.

Unit – VI **Z–TRANSFORMS**

Fundamental difference between continuous and discrete time signals, discrete time signal representation using complex exponential and sinusoidal components, Periodicity of discrete time using complex exponential signal, Concept of Z- Transform of a discrete sequence, Distinction between Laplace, Fourier and Z transforms. Region of convergence in Z-Transform, constraints on ROC for various classes of signals, Inverse Z-transform, properties of Z-transforms.

Signal:

A signal is a pattern of variation of some form. Signals are variables that carry information

Examples of signal include:

Electrical signals -Voltages and currents in a circuit

Acoustic signals - Acoustic pressure (sound) over time

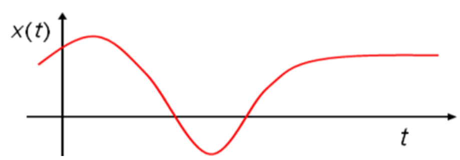
Mechanical signals - Velocity of a car over time

Video signals - Intensity level of a pixel (camera, video) over time.

Mathematically, signals are represented as a function of one or more **independent variables**. For instance a black & white video signal intensity is dependent on x, y coordinates and time t f(x,y,t)

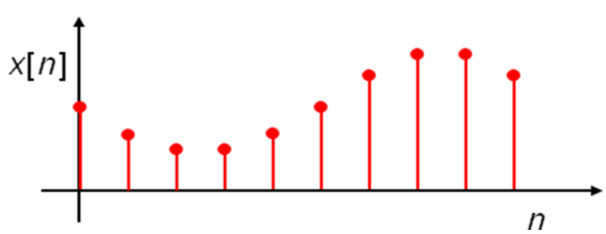
Continuous-Time Signals

Most signals in the real world are continuous time, as the scale is infinitesimally fine. Eg voltage, velocity, Denote by x(t), where the time interval may be bounded (finite) or infinite



Discrete-Time Signals

Some real world and many digital signals are discrete time, as they are sampled. E.g. pixels, daily stock price (anything that a digital computer processes) **,**denote by x[n], where n is an integer value that varies discretely



Signal Properties:

1. **Periodic signals:** a signal is periodic if it repeats itself after a fixed period T, i.e. x(t) = x(t+T) for all t. A sin(t) signal is periodic.

2. **Even and odd signals:** a signal is even if x(-t) = x(t) (i.e. it can be reflected in the axis at zero). A signal is odd if x(-t) = -x(t). Examples are cos(t) and sin(t) signals, respectively.

3. **Exponential and sinusoidal signals**: a signal is (real) exponential if it can be represented as $x(t) = Ce^{at}$. A signal is (complex) exponential if it can be represented in the same form but C and a are complex numbers.

4. **Step and pulse signals:** A pulse signal is one which is nearly completely zero, apart from a short spike, d(t). A step signal is zero up to a certain time, and then a constant value after that time, u(t).

System:

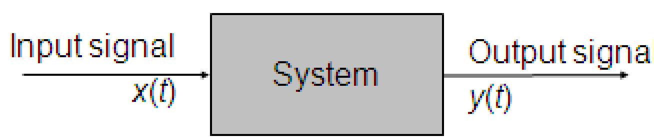
- Systems process input signals to produce output signals

Examples:

1. A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
2. A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
3. A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?

A system takes a signal as an input and transforms it into another signal.



In a very broad sense, a system can be represented as the ratio of the output signal over the input signal. That way, when we "multiply" the system by the input signal, we get the output signal.

Properties of a System:

- **Causal**: a system is causal if the output at a time, only depends on input values up to that time.
- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs
- **Time-invariance**: a system is time invariant if the system's output is the same, given the same input signal, regardless of time.

How Are Signal & Systems Related ?

How to design a system to process a signal in particular ways?

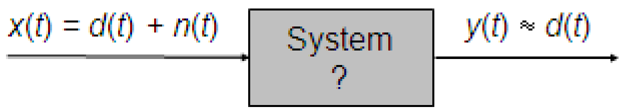
Design a system to restore or enhance a particular signal

1. Remove **high frequency** background communication noise
2. Enhance **noisy** images from spacecraft

Assume a signal is represented as

$$x(t) = d(t) + n(t)$$

Design a system to remove the unknown "noise" component n(t), so that $y(t) \approx d(t)$



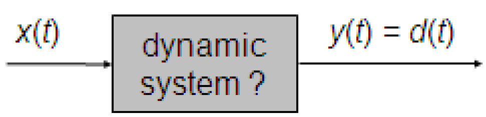
How to design a (dynamic) system to modify or control the output of another (dynamic) system

1. Control an aircraft's altitude, velocity, heading by adjusting throttle, rudder, ailerons
2. Control the temperature of a building by adjusting the heating/cooling energy flow.

Assume a signal is represented as

$$x(t) = g(d(t))$$

Design a system to "invert" the transformation g(), so that y(t) = d(t)



"Electrical" Signal Energy & Power

It is often useful to characterise signals by measures such as **energy** and **power**

For example, the **instantaneous power** of a resistor is:

$$p(t) = v(t)i(t) = \frac{1}{R}v^2(t)$$

and the **total energy** expanded over the interval [t_1, t_2] is:

$$\int_{t_1}^{t_2} p(t)dt = \int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

and the **average energy** is:

$$\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} p(t)dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \frac{1}{R}v^2(t)dt$$

How are these concepts defined for any continuous or discrete time signal?

- Generic Signal Energy and Power

Total energy of a continuous signal x(t) over $[t_1, t_2]$ is:

$$E = \int_{t_1}^{t_2} |x(t)|^2\, dt$$

where |.| denote the magnitude of the (complex) number.

Similarly for a discrete time signal x[n] over $[n_1, n_2]$:

$$E = \sum_{n=n_1}^{n_2} |x[n]|^2$$

By dividing the quantities by $(t_2\text{-}t_1)$ and $(n_2\text{-}n_1\text{+}1)$, respectively, gives the **average power**, P

Note that these are similar to the electrical analogies (voltage), but they are different, both value and dimension.

Energy and Power over Infinite Time:

For many signals, we're interested in examining the power and energy over an infinite time interval (-∞, ∞). These quantities are therefore defined by:

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2\, dt = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

If the sums or integrals do not converge, the energy of such a signal is infinite

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2\, dt$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1}\sum_{n=-N}^{N} |x[n]|^2$$

Two important (sub)classes of signals

1. Finite total energy (and therefore zero average power)
2. Finite average power (and therefore infinite total energy)

Signal analysis over infinite time, all depends on the "tails" (limiting behaviour)

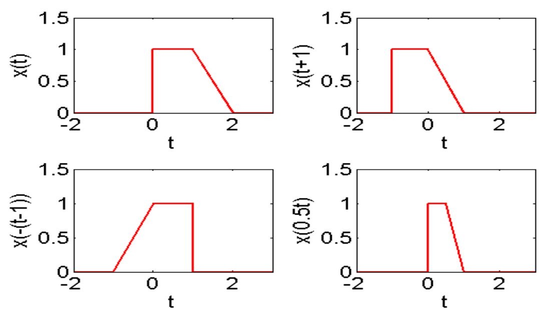
- Time Shift Signal Transformations

A central concept in signal analysis is the transformation of one signal into another signal. Of particular interest are simple transformations that involve a transformation of the time axis only.

A linear **time shift** signal transformation is given by:

$$y(t) = x(at + b)$$

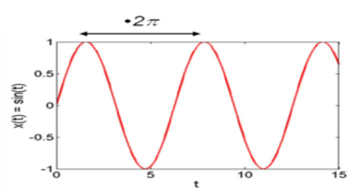
where b represents a signal offset from 0, and the a parameter represents a signal stretching if |a|>1, compression if 0<|a|<1 and a reflection if a<0.



Periodic Signals:

An important class of signals is the class of periodic signals. A periodic signal is a continuous time signal x(t), that has the property $x(t) = x(t+T)$

where T>0, for all t.



Examples:

cos(t+2p) = cos(t), sin(t+2p) = sin(t) Are both periodic with period 2p

NB for a signal to be periodic, the relationship must hold for all t.

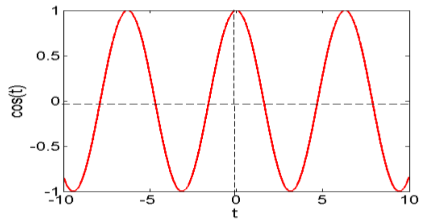
Odd and Even Signals:

An **even** signal is identical to its time reversed signal, i.e. it can be reflected in the origin and is equal to the original: $x(-t) = x(t)$

Examples:

x(t) = cos(t)

x(t) = c

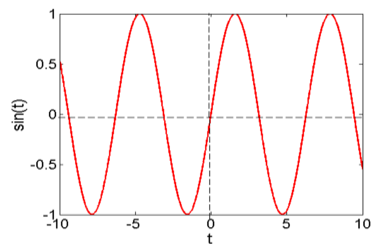


An **odd** signal is identical to its negated, time reversed signal, i.e. it is equal to the negative reflected signal $x(-t) = -x(t)$

Examples:

x(t) = sin(t)

x(t) = t



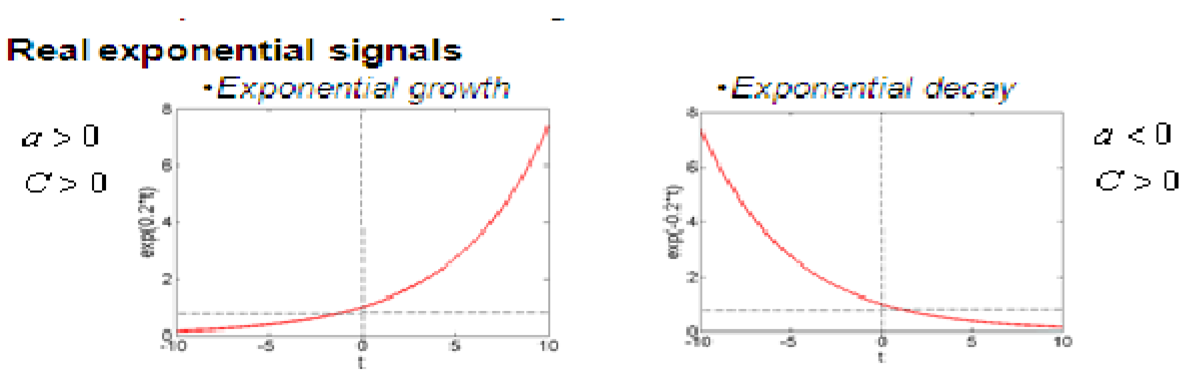
This is important because any signal can be expressed as the sum of an odd signal and an even signal.

Exponential and Sinusoidal Signals:

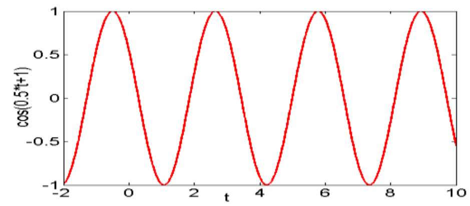
Exponential and sinusoidal signals are characteristic of real-world signals and also from a basis (a building block) for other signals.

A generic **complex exponential signal** is of the form: $x(t) = Ce^{at}$

where C and a are, in general, complex numbers. Lets investigate some special cases of this signal



Periodic Complex Exponential & Sinusoidal Signals:



Consider when a is purely imaginary: $x(t) = Ce^{at}$

By Euler's relationship, this can be expressed as: $e^{j\omega_0 t} = \cos\omega_0 t + j\sin\omega_0 t$

This is a periodic signals because:
$$e^{j\omega_0 (t+T)} = \cos\omega_0 (t+T) + j\sin\omega_0 (t+T)$$
$$= \cos\omega_0 t + j\sin\omega_0 t = e^{j\omega_0 t}$$

when $T=2p/w_0$

A closely related signal is the **sinusoidal signal**: $x(t) = \cos(\omega_0 t + \phi)$

We can always use:
$$x(t) = Ce^{j\omega_0 t}$$
$$A\cos(\omega_0 t + \phi) = A\Re\left(e^{j(\omega_0 t + \phi)}\right)$$
$$A\sin(\omega_0 t + \phi) = A\Im\left(e^{j(\omega_0 t + \phi)}\right)$$

Exponential & Sinusoidal Signal Properties:

Periodic signals, in particular complex periodic and sinusoidal signals, have infinite total energy but finite average power.

Consider energy over one period:
$$E_{period} = \int_0^{T_0} \left|e^{j\omega_0 t}\right|^2 dt$$
$$= \int_0^{T_0} 1 dt = T_0$$

Therefore: $E_\infty = \infty$

Average power: $P_{period} = \frac{1}{T_0} E_{period} = 1$

Complex Exponential Signals:

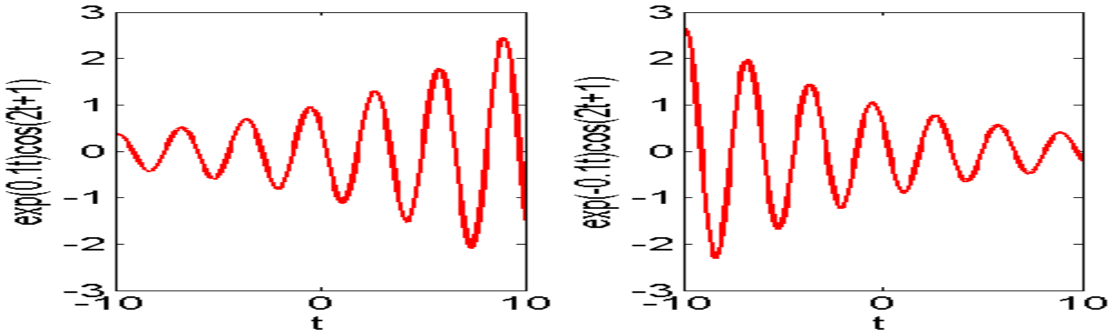
So far, considered the real and periodic complex exponential

Now consider when C can be complex. Let us express C is polar form and a in rectangular form:
$$C = |C|e^{j\phi}$$
$$a = r + j\omega_0$$

So
$$Ce^{at} = |C|e^{j\phi}e^{(r+j\omega_0)t} = |C|e^{rt}e^{j(\omega_0+\phi)t}$$

Using Euler's relation
$$Ce^{at} = |C|e^{j\phi}e^{(r+j\omega_0)t} = |C|e^{rt}\cos((\omega_0+\phi)t) + j|C|e^{rt}\sin((\omega_0+\phi)t)$$

These are **damped sinusoids**



Discrete Unit Impulse and Step Signals:

The discrete **unit impulse signal** is defined:

$$x[n] = \delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

Useful as a **basis** for analyzing other signals

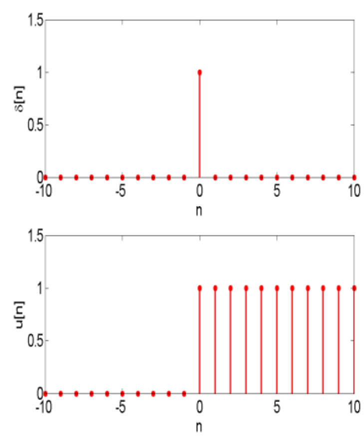
The discrete **unit step signal** is defined:

$$x[n] = u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

Note that the unit impulse is the first difference (derivative) of the step signal

$$\delta[n] = u[n] - u[n-1]$$

Similarly, the unit step is the running sum (integral) of the unit impulse.



Continuous Unit Impulse and Step Signals:

The continuous **unit impulse signal** is defined:

$$x(t) = \delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \end{cases}$$

Note that it is discontinuous at t=0

The arrow is used to denote area, rather than actual value

Again, useful for an infinite basis

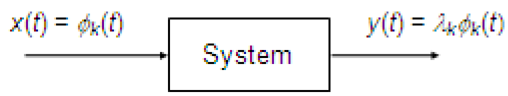
The continuous **unit step signal** is defined:

$$x(t) = u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$$x(t) = u(t) = \begin{cases} 0 & t < 0 \\ 1 & t > 0 \end{cases}$$

Why is Fourier Theory Important ?

For a particular system, what signals $f_k(t)$ have the property that:



Then $f_k(t)$ is an eigenfunction with eigenvalue l_k

If an input signal can be decomposed as

$$x(t) = \Sigma_k a_k \phi_k(t)$$

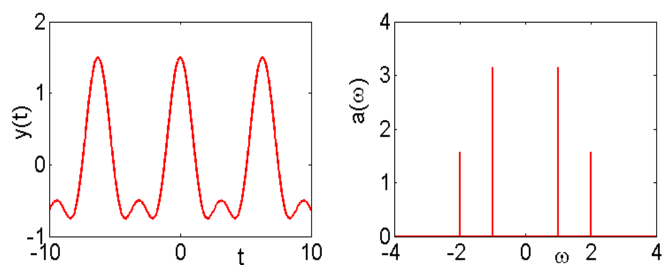
Then the response of an LTI system is

$$y(t) = \Sigma_k a_k \lambda_k \phi_k(t)$$

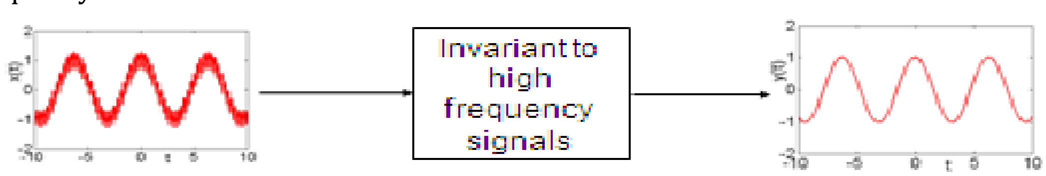
For an LTI system, $f_k(t) = e^{st}$ where $s \in C$, are eigenfunctions.

Fourier transforms map a time-domain signal into a frequency domain signal

Simple interpretation of the frequency content of signals in the frequency domain (as opposed to time).



Design systems to filter out high or low frequency components. Analyse systems in frequency domain.



If F{x(t)} = X(jw) w is the frequency

Then F{x'(t)} = jwX(jw)

So solving a differential equation is transformed from a calculus operation in the time domain into an algebraic operation in the frequency domain (see Laplace transform)

Example $$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+3y=0$$

becomes $$-\omega^2Y(j\omega)+j2\omega Y(j\omega)+3Y(j\omega)=0$$

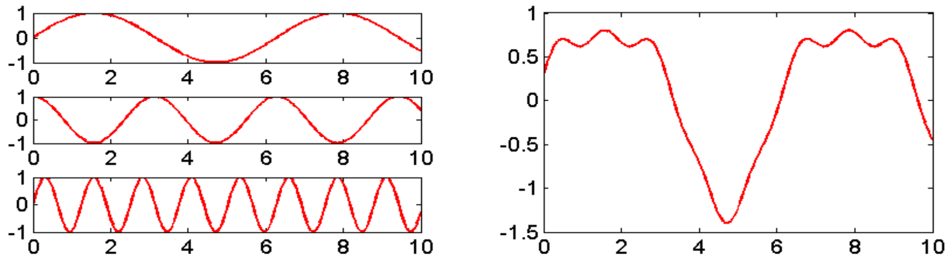
and is solved for the roots w (N.B. complementary equations):

$$-\omega^2+j2\omega+3=0$$

and we take the inverse Fourier transform for those w.

Fourier Series and Fourier Basis Functions:

The theory derived for LTI convolution, used the concept that any input signal can represented as a linear combination of shifted impulses (for either DT or CT signals) . These are known as continuous-time Fourier series. The bases are scaled and shifted sinusoidal signals, which can be represented as complex exponentials.



Periodic Signals & Fourier Series:

A periodic signal has the property x(t) = x(t+T), T is the fundamental period, w_0 = 2p/T is the fundamental frequency. Two periodic signals include:

$$x(t)=\cos(\omega_0 t)$$
$$x(t)=e^{j\omega_0 t}$$

For each periodic signal, the Fourier basis the set of harmonically related complex exponentials: $$\phi_k(t)=e^{jk\omega_0 t}=e^{jk(2\pi/T)t} \qquad k=0,\pm1,\pm2,\ldots$$

Thus the Fourier series is of the form: $$x(t)=\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}=\sum_{k=-\infty}^{\infty}a_k e^{jk(2\pi/T)t}$$

k=0 is a constant

k=+/-1 are the fundamental/first harmonic components

k=+/-N are the N^{th} harmonic components

Fourier Series Representation of a CT Periodic Signal:

Given that a signal has a Fourier series representation, we have to find $\{a_k\}_k$. Multiplying through by $e^{-jn\omega_0 t}$

$$x(t)e^{-jn\omega_0 t}=\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}e^{-jn\omega_0 t}$$

$$\int_0^T x(t)e^{-jn\omega_0 t}dt=\int_0^T\sum_{k=-\infty}^{\infty}a_k e^{j(k-n)\omega_0 t}dt$$

$$=\sum_{k=-\infty}^{\infty}a_k\int_0^T e^{j(k-n)\omega_0 t}dt$$

Using Euler's formula for the complex exponential integral

$$\int_0^T e^{j(k-n)\omega_0 t}dt = \int_0^T \cos((k-n)\omega_0 t)dt + j\int_0^T \sin((k-n)\omega_0 t)dt$$

It can be shown that

$$\int_0^T e^{j(k-n)\omega_0 t}dt = \begin{cases} T & k = n \\ 0 & k \neq n \end{cases}$$

Therefore $$a_n = \tfrac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$$

which allows us to determine the coefficients. Also note that this result is the same if we integrate over any interval of length T (not just [0,T]), denoted by

$$a_n = \tfrac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt$$

$$\int_T |x(t)|dt < \infty$$

To summarise, if x(t) has a Fourier series representation, then the pair of equations that defines the Fourier series of a periodic, continuous-time signal:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$$

$$a_k = \tfrac{1}{T}\int_T x(t)e^{-jk\omega_0 t}dt = \tfrac{1}{T}\int_T x(t)e^{-jk(2\pi/T)t}dt$$

Example 1: Fourier Series $\sin(w_0 t)$

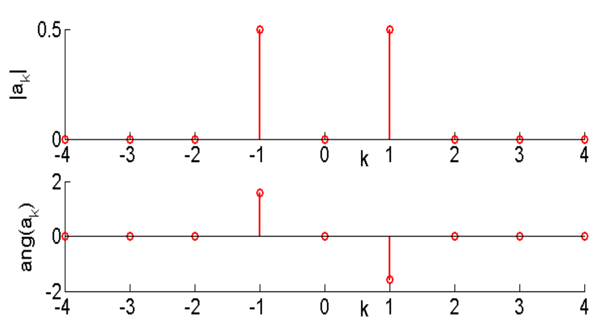
The fundamental period of $\sin(w_0 t)$ is w_0

By inspection we can write:

$$\sin(\omega_0 t) = \tfrac{1}{2j}e^{j\omega_0 t} - \tfrac{1}{2j}e^{-j\omega_0 t}$$

So $a_1 = 1/2j$, $a_{-1} = -1/2j$ and $a_k = 0$ otherwise

The magnitude and angle of the Fourier coefficients are:



Convergence of Fourier Series:

Not every periodic signal can be represented as an infinite Fourier series, however just about all interesting signals can be (note that the step signal is discontinuous)

The **<u>Dirichlet conditions</u>** are necessary and sufficient conditions on the signal.

1. Over any period, x(t) must be absolutely integrable. $\int_T |x(t)|dt < \infty$

2. In any finite interval, x(t) is of bounded variation; that is there is no more than a finite number of maxima and minima during any single period of the signal
3. In any finite interval of time, there are only a finite number of discontinuities. Further, each of these discontinuities are finite.

Complex exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}, \; t_0 \le t \le t_0 + T_0$$

$$\text{where } X_n = \frac{1}{T_0} \int_{T_0} x(t) e^{-jn\omega_0 t}$$

Trignometic Form:

The complex exponential Fourier series can be arranged as follows

$$x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}$$

$$= X_0 + \sum_{n=1}^{\infty} \left[X_n e^{jn\omega_0 t} + X_{-n} e^{-jn\omega_0 t} \right]$$

Description of Fourier Series:

FourierSeries.ppt

Multiple Choice Questions

1. **Which one most appropriate dynamic system?**

A. y(n) + y(n - 1) + y(n + 1)

B. y(n) + y(n - 1)

C. y(n) = x(n)

D. y(n) + y(n - 1) + y(n + 3) = 0

Option A. Because present output of y(n) depend upon past y(n - 1) and future y(n + 1).

2 . **An energy signal has G(f) = 10. Its energy density spectrum is**

A. 10 **B.** 100

C. 50 **D.** 20

Option B. Energy density spectrum = |G(f)|² = |10|² = 100.

3 A voltage V(t) is a Gaussian ergodic random process with a mean of zero and a variance of 4 volt². If it is measured by a dc meter. The reading will be

A. 0

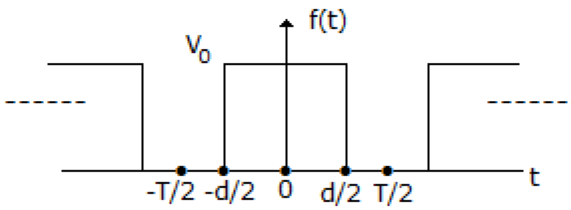
B. 4

C. 2

D. 2

Option A. DC meter reads mean value.

4 In the given figure the ratio T/d is the duty factor.



A. True **B.** False

Option B. Duty factor is $\frac{d}{t}$.

5 . The period of the function cos $\frac{\pi}{4}(t-1)$ is

A. $\frac{1}{8}s$

B. 8s

C. 4s

D. $\frac{1}{4}s$

Option B. $f = \frac{1}{8}$ Hz and T = 8s.

6 . Which one is a linear system?

A. y[n] = x[n] x x[n - 1]

B. $y[n] = x[n] + x[n - 10]$

C. $y[n] = x^2[n]$

D. (a) and (c)

<u>Option B</u>

7 . A voltage wave having 5% fifth harmonic content is applied to a series RC circuit. The percentage fifth harmonic content in the current wave will be

A. 5%

B. more than 5%

C. less than 5%

D. equal or more than 5%

<u>Option B</u> **<u>$I_C = V(j\omega C)$.</u>**

8 . δ(t) is a

A. energy signal

B. power signal

C. neither energy nor power

D. None

<u>Option B . Because Unit step is a Power signal. So By trignometric identifies d(t) also power.</u>

$$\delta(t) = \frac{d\,u(t)}{dt}.$$

9. The analog signal given below is sampled by 600 samples per second for m(t) = 3 sin 500 ⍰t + 2 sin 700 ⍰t then folding frequency is

A. 500 Hz

B. 700 Hz

C. 300 Hz

D. 1400 Hz

<u>Option C</u>

10. The signal defined by the equations f(t) = 0 for t < 0, f(t) = E for 0 ≤ t ≤ a and f(t) = 0

for t > a is

A. a step function

B. a pulse function

C. a shifted step function originating at t = a

D. none of the above

<u>Option B. It is a pulse lasting for t = a.</u>

Fill in the blanks

1. Area under the impulse signal is --------------- **(1)**
2. step signal is passed through integrator then output will be**……………(ramp signal)**
3. output depends present input and past input then system is **……………(causal system**)
4. The signal is present in only right side t>0 then signal is called**……………(causal signal)**
5. example of periodic signal is………….(**Impulse signal)**
6. impulse signal is passed through the integrator then output will be**…….(step signal)**
7. give an example of non causal system**………(log(x(t+2)))**
8. give an example of passive element of dynamic system is ……….. **(R,L&c)**
9. BIBO then system **…………… (Stable)**
10. give an example of invertible system is**………(y(t)=2x(t))**

Review Questions

Objective type questions:

1. Define Signal.
2. Define system.
3. What are the major classifications of the signal?
4. Define discrete time signals and classify them.
5. Define continuous time signals and classify them.
6. Define discrete time unit step &unit impulse.
7. Define continuous time unit step and unit impulse.
8. Define unit ramp signal.
9. Define periodic signal and non periodic signal.

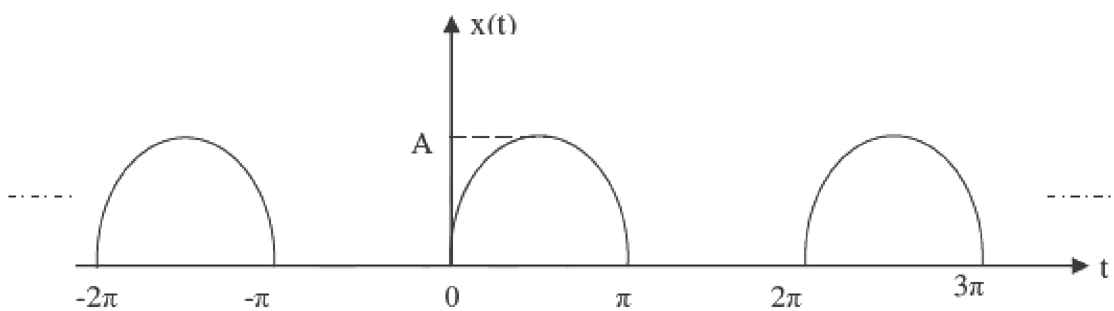
10. Define even and odd signal?
11. Define Energy and power signal.
12. Define unit pulse function.
13. Define continuous time complex exponential signal.
14. What is continuous time real exponential signal?
15. What is continuous time growing exponential signal?
16. What is continuous time decaying exponential?
17. What are the types of Fourier series?
18. Write down the exponential form of the Fourier series representation of a periodic signal?
19. Write down the trigonometric form of the fourier series representation of a periodic signal?
20. Write short notes on dirichlets conditions for Fourier series.
21. State Time Shifting property in relation to fourier series.
22. State parseval's theorem for continuous time periodic signals.

Analytical Questions:

1. Need of Dirichlet Conditions.
2. Justify how the energy of a Power signal is infinite.
3. Justify how the Power of a Energy signal is zero.

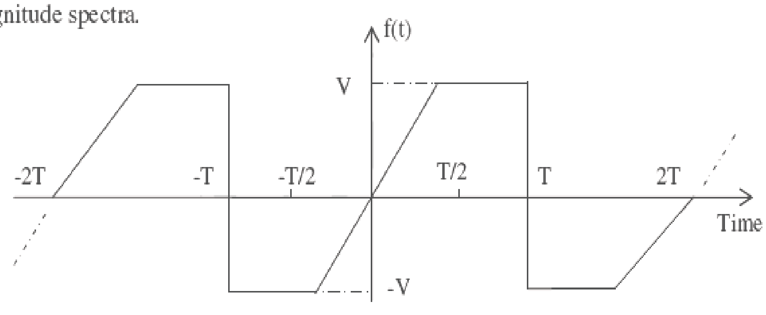
Essay Questions

1. Find the expression for mean square error using the expression of a function using orthogonal signal space.
2. Obtain the relations between the coefficients of trigonometric Fourier series and Exponential Fourier series.
3. Find the Exponential Fourier series expansion of the half wave rectified sine wave shown below.



4. Write Dirichlet's conditions to obtain Fourier series representation of any signal.
5. Find the Trigonometric Fourier series for the periodic ware from shown below and draw it

magnitude spectra.



Skill Building Exercises/ Assignments

1. Find the Fourier series representation and sketch the amplitude and phase spectrum for the signal x(n) = 5 + sin(nπ/2) + cos(nπ/4) .
2. Find the even and odd components of the signal x(t) = cost + sin t + cost sint.

3. Derive the expression for component vector of approximating the function f1(t) over f2(t) and also prove that the component vector becomes zero if the f1(t) and f2(t) are orthogonal.
4. A rectangular function f(t) is defined by
   i. $f(t)=\begin{cases} 1 & 0<t<\pi \\ -1 & \pi<t<2\pi \end{cases}$

Approximate this function by a waveform sin(t) over the interval (0, 2π) such that the mean square error is minimum.

Previous Questions (Asked by JNTUK from the concerned Unit)

1. A rectangular function defined by
   a. f(t) = 1; 0<t<
      i. -1; π<t<2π
   b. Approximate above rectangular function by a single sinusoid "sin t", evaluate mean square error in this approximation . Also show what happens when more number of sinusoidals are used for approximation.

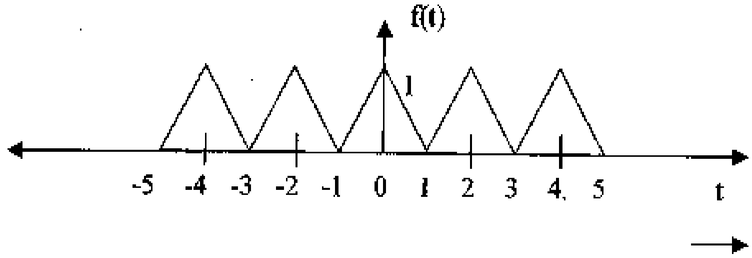
2. Define and sketch the following elementary signals
   a. unit impulse signal ii) unit step signal iii) signum function
3. Discuss the analogy between vectors and signals and hence explain orthogonalvector space and orthogonal signal spaces. Explain the condition of orthogonalitybetween two signal f1(t) & f2(t).
4. State different properties of Fourier series.

5. With regard to Fourier series representation, justify the following statements
   a. Odd functions have only sine term
   b. Even functions have no sine term
   c. Functions with half wave symmetry have only odd harmonics.

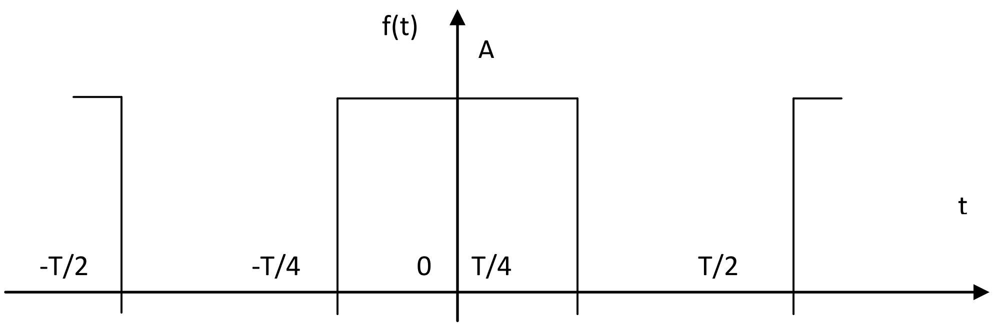
6. A rectangular function defined as
   f(t) = A; 0<t<π/2
   -A; π/2<t<3π/2
   A; 3π/2<t<2π
   Approximate above function by Acost between the intervals (0, 2π) such that mean square error is minimum.

7. Show that the functions sin nwot and sin mwot are orthogonal to each otherfor all integer values of m and n.

8. Find the exponential Fourier series and plot the magnitude and phase spectrum of the following triangular wave form.



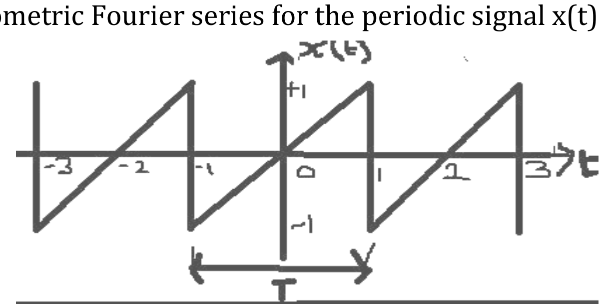
9. Obtain the Fourier components of the periodics square wave signal which is symmterical with respect to vertical axis at time t=0 as shown in the figure.



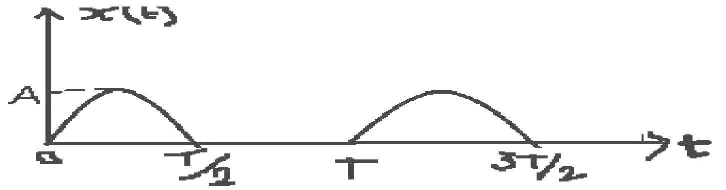
10. Explain how a function can be approximated by a set of orthogonal functions. Discuss the concept of trigonometric Fourier series and derive the expressions for coefficients.
11. Define orthogonal functions. Give some examples of orthogonal functions.
12. Obtain the condition under which two signals f1 (t) and f2 (t) are said to be orthogonal to each other. Hence prove that cos $n\omega_0 t$ and cos $m\omega_0 t$ are orthogonal over any interval
13. A rectangular function defined by f(t)= 1; $0 < t <$ -1;
    $\pi < t < 2\pi$

approximate the above function by a single sinusoid sin t, Evaluate mean square error in this approximation. Also show what happens when more number of sinusoidal are used for approximations.

14. Define and sketch the following elementary continuous time signals.
    i) Unit impulse signal
    ii) Signum function
    iii) unit step function
15. Find the trigonometric Fourier series for the periodic signal x(t) shown below.



16. Obtain the trigonometric Fourier series for the half wave rectified sine wave as given below.
17.



Explain about complex Fourier spectrum.

Gate Questions (Where relevant)

1. The Fourier Series of an odd periodic function, contains only

   a. Odd harmonics

   b. even harmonics

   c. cosine terms

   d. sine terms GATE 1994

2. 2. The RMS value of a rectangular wave of period T, having a value of +V for a duration, T1(<T) and –V for the duration, T-T1 = T2 equals

   a. V

   b. (T1-T2)/T V

   c. V/√2

   d. T1 /T V GATE 1995

3. The Trigonometric Fourier Series of a periodic time function can have only

   a. cosine terms

   b. sine terms

   c. cosine & sine terms

   d. DC& cosine terms GATE 1998

4. Which of the following cannot be the Fourier Series expansion of a periodic signal?

   a. x(t) = 2 cos t +3 cos 3t

   b. x(t) = 2 cos πt +7 cos t

   c. x(t) = cos t +0.5

   d. x(t) = 2 cos 1.5πt + sin 3.5 πt GATE 2002

5. Choose the function f(t); -∞ < t < +∞, for which a Fourier Series cannot be defined

a. 3 sin (25t)

b. 4 cos(20t+3) +3 sin (10t)

c. exp(-|t|)sin (25t)

d. 1 GATE 2005

6. The Trigonometric Fourier Series of an even function of time does not have

a. d.c. terms

b. cosine terms

. sine terms

d. odd harmonic terms GATE 1996,2011

7. Trigonometric Fourier Series of a real periodic function has only

a. cosine terms if it is even

b. sine terms if it is even

c. cosine terms if it is odd

d. sine terms if it is odd Which of the above statements are correct?

a. P and S b. P and R c. Q and S d. Q and R GATE 2009

8. A function is given by f(t) = sin2t + cos 2t. which of the following is true? GATE 2009

1.1. f has frequency components at 0 and 1/2π Hz

1.2. **f has frequency components at 0 and 1/π Hz**

1.3. c. f has frequency components at 1/2π&1/π Hz

1.4. f has frequency components at 0,1/2π &1/π Hz

9. A half – wave rectified sinusoidal waveform has a peak voltage of 10 V. Its average value and the peak value of the fundamental component are respectively given by:

a. 20/ π V, 10/√2 V

b. 10/π V, 10/√2 V

c. 10/ π V, 5 V

. 20/ π V, 5 V GATE 1987

10. Which of the following signals is/are periodic? GATE 1992

Deriving Fourier transform from Fourier series, Fourier transform of arbitrary signal, Fourier transform of standard signals, Fourier transform of periodic signals, Properties of Fourier transforms, Fourier transforms involving impulse function and Signum function, Introduction to Hilbert Transform.

a. s(t) = cos 2t + cos 3t + cos 5t

b. s(t) = exp(j8πt)

c. s(t) = exp(-7t)sin 10πt

d. s(t) = cos 2t cos 4t

11. The PSD and the power of a signal g(t), are respectively, Sg(w), Pg. The PSD and the power of the signal a g(t) are, respectively,

a. a2Sg(w), a2 Pg

b. a2Sg(w), a Pg

c. aSg(w), a2 Pg

d. aSg(w), a Pg GATE 2001

Unit – 2

Convergence of Fourier Series

Not every periodic signal can be represented as an infinite Fourier series, however just about all interesting signals can be (note that the step signal is discontinuous) the Dirichlet conditions are necessary and sufficient conditions on the signal.

Condition 1. Over any period, x(t) must be absolutely integrable $\int_T |x(t)| dt < \infty$

Condition 2. In any finite interval, x(t) is of bounded variation; that is there is no more than a finite number of maxima and minima during any single period of the signal

Condition 3. In any finite interval of time, there are only a finite number of discontinuities. Further, each of these discontinuities are finite.

Fourier Series to Fourier Transform

For periodic signals, we can represent them as linear combinations of harmonically related complex exponentials To extend this to non-periodic signals, we need to consider aperiodic signals as periodic signals with infinite period. As the period becomes infinite, the corresponding frequency components form a continuum and the Fourier series sum becomes an integral (like the derivation of CT convolution) Instead of looking at the coefficients a harmonically –related Fourier series, we'll now look at the **Fourier transform** which is a **complex valued function** in the frequency domain

Definition of the Fourier Transform

We will be referring to functions of time and their Fourier transforms. A signal x(t) and its Fourier transform X(jw) are related by the **Fourier transform synthesis** and **analysis** equations

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = F\{x(t)\}$$

And

$$x(t) = \tfrac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = F^{-1}\{X(j\omega)\}$$

We will refer to x(t) and X(jw) as a **Fourier transform pair** with the notation $x(t) \overset{F}{\leftrightarrow} X(j\omega)$

As previously mentioned, the transform function X() can roughly be thought of as a continuum of the previous coefficients

A similar set of Dirichlet convergence conditions exist for the Fourier transform, as for the Fourier series (T=(- ∞,∞))

Decaying Exponential :

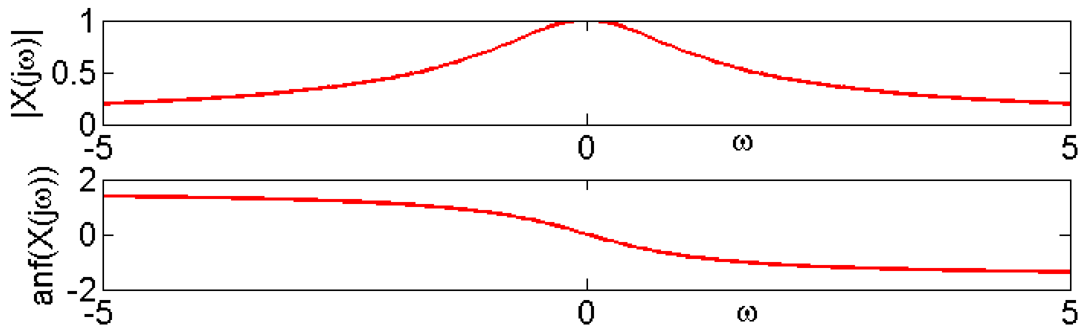
Consider the (non-periodic) signal $x(t) = e^{-at} u(t) \qquad a > 0$

Then the Fourier transform is:

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{-j\omega t} dt = \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{1}{-(a+j\omega)} e^{-(a+j\omega)t} \Big|_0^{\infty}$$

1



Single Rectangular Pulse:

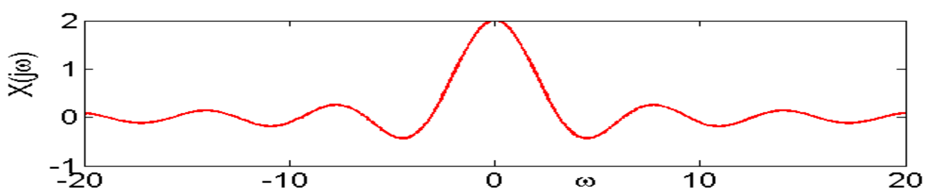
Consider the non-periodic rectangular pulse at zero $x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| \geq T_1 \end{cases}$

The Fourier transform is:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{-T_1}^{T_1} e^{-j\omega t}dt$$

$$= \frac{1}{-j\omega} e^{-j\omega t}\Big|_{-T_1}^{T_1}$$

$$= \frac{2\sin(\omega T_1)}{\omega}$$



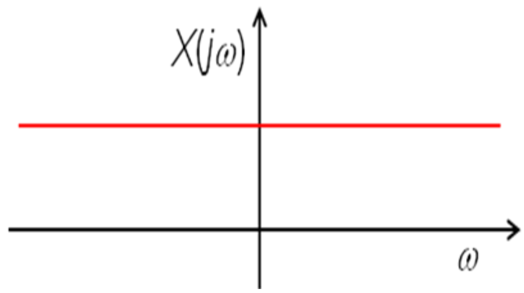
Impulse Signal:

The Fourier transform of the impulse signal can be calculated as follows:

$$x(t) = \delta(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}dt = 1$$

Therefore, the Fourier transform of the impulse function has a constant contribution for **all frequencies**



Periodic Signals:

A periodic signal violates condition 1 of the Dirichlet conditions for the Fourier transform to exist

However, lets consider a Fourier transform which is a single impulse of area 2p at a particular (harmonic) frequency w=w_0. $X(j\omega) = 2\pi\delta(\omega - \omega_0)$

The corresponding signal can be obtained by:

$$x(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)e^{j\omega t}d\omega = e^{j\omega_0 t}$$

which is a (complex) sinusoidal signal of frequency w_0. More generally, when

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

Then the corresponding (periodic) signal is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

The Fourier transform of a periodic signal is a train of impulses at the harmonic frequencies with amplitude 2pa_k.

Fourier Transform Properties and Examples:

Basis functions : Concept of basis function. Fourier series representation of time functions. **Fourier transform and its properties**. **Examples, transform of simple time functions**.

Properties of a Fourier transform

- Linearity
- Time shifts
- Differentiation and integration
- Convolution in the frequency domain
-

A signal x(t) and its Fourier transform X(jw) are related by

$$x(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

This is denoted by:

$$x(t) \overset{F}{\leftrightarrow} X(j\omega)$$

For example (1):

$$e^{-at}u(t) \overset{F}{\leftrightarrow} \frac{1}{a+j\omega}$$

Remember that the Fourier transform is a **density function**, you must integrate it, rather than summing up the discrete Fourier series components

Linearity of the Fourier Transform

If
$$x(t) \overset{F}{\leftrightarrow} X(j\omega)$$

And
$$y(t) \overset{F}{\leftrightarrow} Y(j\omega)$$

Then
$$ax(t) + by(t) \overset{F}{\leftrightarrow} aX(j\omega) + bY(j\omega)$$

This follows directly from the definition of the Fourier transform (as the integral operator is linear). It is easily extended to a linear combination of an arbitrary number of signals

Time Shifting

If
$$x(t) \overset{F}{\leftrightarrow} X(j\omega)$$

Then
$$x(t-t_0) \overset{F}{\leftrightarrow} e^{-j\omega t_0} X(j\omega)$$

Proof
$$x(t) = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Now replacing t by t-t_0
$$\begin{aligned} x(t-t_0) &= \tfrac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega(t-t_0)} d\omega \\ &= \tfrac{1}{2\pi}\int_{-\infty}^{\infty} \left(e^{-j\omega t_0} X(j\omega)\right) e^{j\omega t} d\omega \end{aligned}$$

Recognising this as

$$F\{x(t-t_0)\} = e^{-j\omega t_0} X(j\omega)$$

A signal which is shifted in time does not have its Fourier transform magnitude altered, only a shift in phase.

Linearity & Time Shift

Consider the signal (linear sum of two time shifted steps)

$$x(t) = 0.5x_1(t-2.5) + x_2(t-2.5)$$

where $x_1(t)$ is of width 1, $x_2(t)$ is of width 3, centred on zero.

Using the rectangular pulse example

$$X_1(j\omega) = {}^{2\sin(\omega/2)}\!\!\diagup_{\omega}$$

$$X_2(j\omega) = {}^{2\sin(3\omega/2)}\!\!\diagup_{\omega}$$

Then using the **linearity** and **time shift** Fourier transform properties

$$X(j\omega) = e^{-j5\omega/2}\left({}^{(\sin(\omega/2)+2\sin(3\omega/2))}\!\!\diagup_{\omega}\right)$$

Differentiation & Integration

By differentiating both sides of the Fourier transform synthesis equation:

$$\frac{dx(t)}{dt} = \tfrac{1}{2\pi}\int_{-\infty}^{\infty} j\omega X(j\omega)e^{j\omega t}\,d\omega$$

Therefore:

$$\frac{dx(t)}{dt} \overset{F}{\leftrightarrow} j\omega X(j\omega)$$

This is important, because it replaces **differentiation** in the **time domain** with **multiplication** in the **frequency domain**.

Integration is similar:

$$\int_{-\infty}^{t} x(\tau)d\tau = \frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$$

The impulse term represents the dc or average value that can result from integration

Fourier Transform of a Step Signal:

Lets calculate the Fourier transform X(jw)of x(t) = u(t), making use of the knowledge that:

$$g(t) = \delta(t) \overset{F}{\leftrightarrow} G(j\omega) = 1$$

and noting that:

$$x(t) = \int_{-\infty}^{t} g(\tau)d\tau$$

Taking Fourier transform of both sides

$$X(j\omega) = \frac{G(j\omega)}{j\omega} + \pi G(0)\delta(\omega)$$

using the integration property. Since G(jw) = 1:

$$X(j\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

We can also apply the differentiation property in reverse

$$\delta(t) = \frac{du(t)}{dt} \overset{F}{\leftrightarrow} j\omega\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) = 1$$

Convolution in the Frequency Domain:

With a bit of work (next slide) it can show that:

$$y(t) = h(t) * x(t) \overset{F}{\leftrightarrow} Y(j\omega) = H(j\omega)X(j\omega)$$

Therefore, to apply **convolution in the frequency domain**, we just have to multiply the two functions.

To solve for the differential/convolution equation using Fourier transforms:

1. Calculate Fourier transforms of x(t) and h(t)
2. Multiply H(jw) by X(jw) to obtain Y(jw)
3. Calculate the inverse Fourier transform of Y(jw)

Multiplication in the frequency domain corresponds to convolution in the time domain and vice versa.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Convolution Property:

Taking Fourier transforms gives:

$$Y(j\omega) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau\right)e^{-j\omega t}dt$$

Interchanging the order of integration, we have

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau)\left(\int_{-\infty}^{\infty} h(t-\tau)e^{-j\omega t}dt\right)d\tau$$

By the time shift property, the bracketed term is $e^{-jwt}H(jw)$, so

$$Y(j\omega) = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}H(j\omega)d\tau$$

$$= H(j\omega)\int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}d\tau$$

$$= H(j\omega)X(j\omega)$$

Solving an ODE:

Consider the LTI system time impulse response :

$$h(t) = e^{-bt}u(t) \qquad b > 0$$

to the input signal

$$x(t) = e^{-at}u(t) \qquad a > 0$$

Transforming these signals into the frequency domain $H(j\omega) = \dfrac{1}{b + j\omega}, \quad X(j\omega) = \dfrac{1}{a + j\omega}$

and the frequency response is

$$Y(j\omega) = \frac{1}{(b + j\omega)(a + j\omega)}$$

to convert this to the time domain, express as partial fractions:

$$Y(j\omega) = \frac{1}{b-a}\left(\frac{1}{(a + j\omega)} - \frac{1}{(b + j\omega)}\right)$$

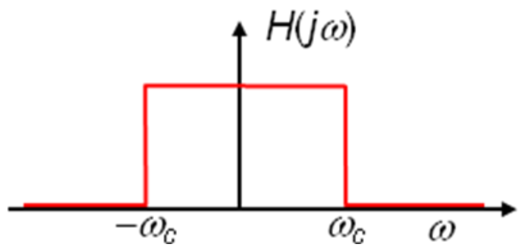
Therefore, the time domain response is:

$$y(t) = \tfrac{1}{b-a}\left(e^{-at}u(t) - e^{-bt}u(t)\right)$$

Designing a Low Pass Filter:

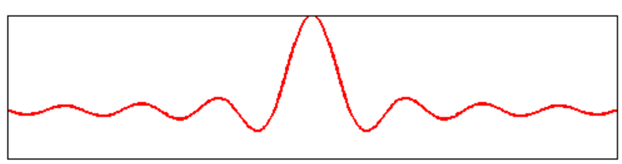
Lets design a low pass filter:

$$H(j\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$



The impulse response of this filter is the inverse Fourier transform

$$h(t) = \tfrac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\, d\omega = \frac{\sin(\omega_c t)}{\pi t}$$



which is an ideal low pass filter

- Non-causal (how to build)
- The time-domain oscillations may be undesirable

How to approximate the frequency selection characteristics?

Consider the system with impulse response:

$$e^{-at}u(t) \overset{F}{\leftrightarrow} \frac{1}{a + j\omega}$$

Causal and non-oscillatory time domain response and performs a degree of low pass filtering

Fourier series and **Fourier transform** is used to represent **periodic** and **non-periodic** signals in the **frequency domain**, respectively. Looking at signals in the Fourier domain allows us to understand the frequency response of a system and also to design systems with a particular frequency response, such as filtering out high frequency signals. You'll need to complete the exercises to work out how to calculate the Fourier transform (and its inverse) and evaluate the frequency content of a signal. The Fourier transform is widely used for designing **filters**. You can design systems with reject high frequency noise and just retain the low frequency components. This is natural to describe in the frequency domain.

Important properties of the Fourier transform are:

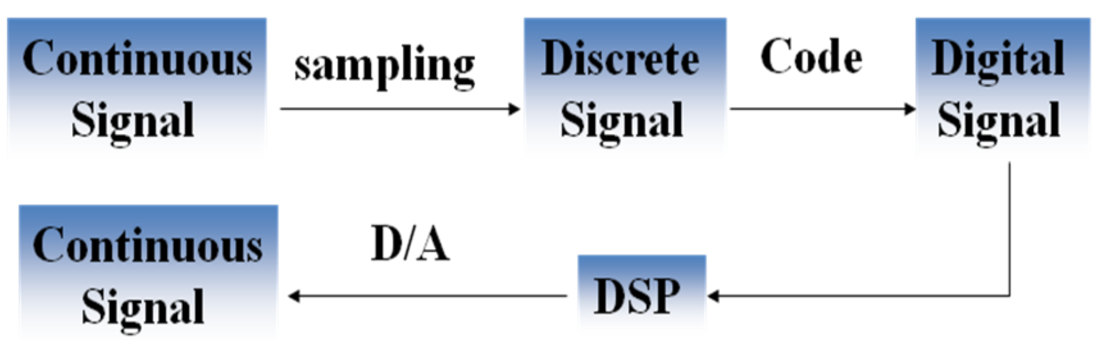
1. Linearity and time shifts

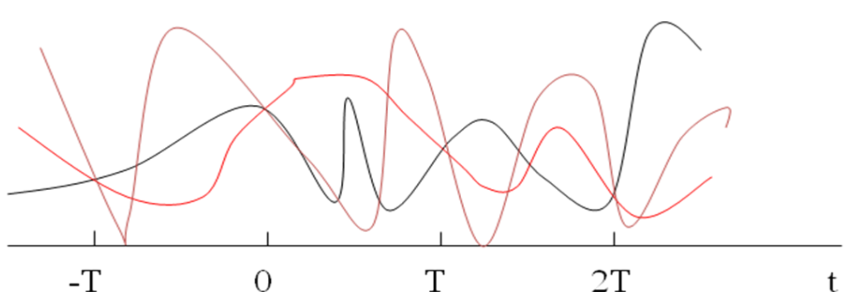
2. Differentiation

3. Convolution

Some operations are simplified in the frequency domain, but there are a number of signals for which the Fourier transform do not exist – this leads naturally onto **Laplace transforms.**

Sampling Theorem and its Reconstruction:



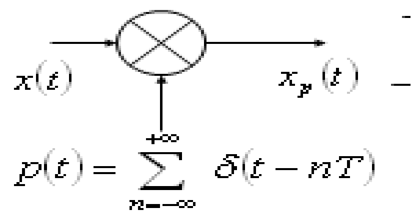


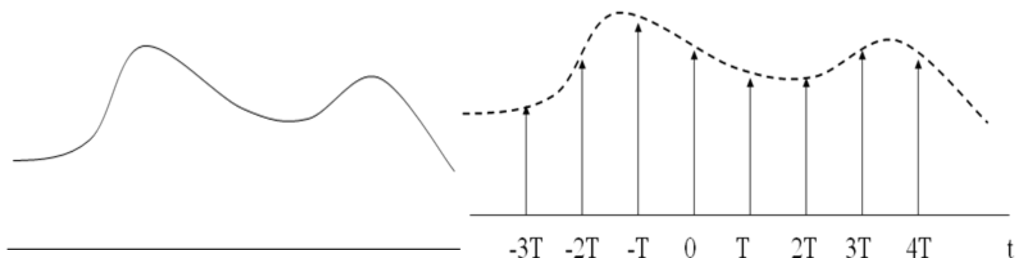
$$x_1(nT) = x_2(nT) = x_3(nT)$$

$$x_1(t) \neq x_2(t) \neq x_3(t)$$

The Sampling Theorem

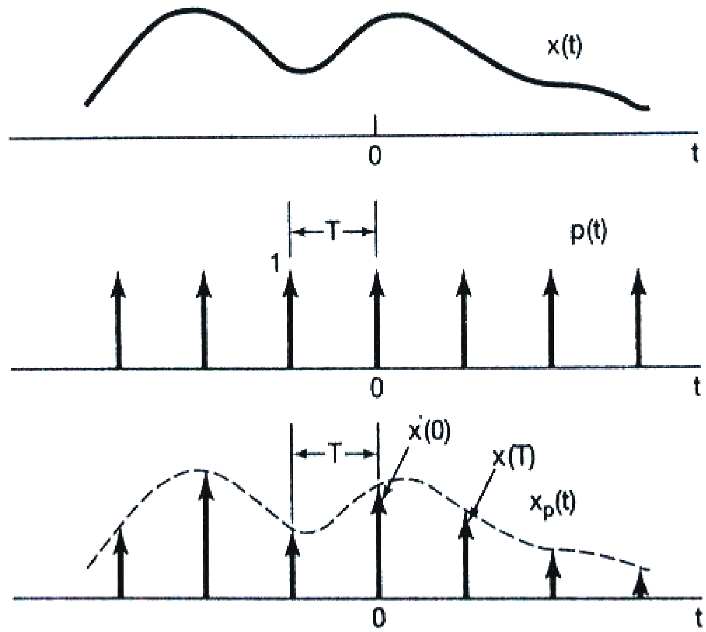
Impulse-Train Sampling





Time domain:

$$x_p(t) = x(t) \cdot \delta_T(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$



Frequency Domain:

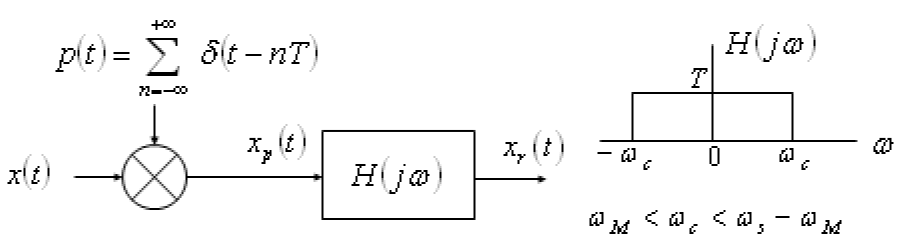
$$x(t) \xleftrightarrow{F} X(j\omega)$$

$$p(t) \xleftrightarrow{F.S.} a_k = \frac{1}{T} \quad (Periodic\ \ signal)$$

$$p(t) \xleftrightarrow{F} P(j\omega) = \sum_{k=-\infty}^{+\infty} 2\pi a_k \delta(\omega - k\omega_s) = \sum_{k=-\infty}^{+\infty} \omega_s \delta(\omega - k\omega_s)$$

$$x_p(t) \xleftrightarrow{F} X_p(j\omega) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(\omega - k\omega_s)$$

The reconstruction of the signal



$$x(t) = \sum_{n=-\infty}^{+\infty} x(nT) S_a\left[\omega_H\left(t - nT\right)\right]$$

Multiple Choice Questions:

1 If $H(f) = \dfrac{y(t)}{x(t)}$ then for this to be true x(t) is

A. $\exp(j2\pi f/t)$

B. $\exp(-j2\pi f/t)$

C. $\exp(j2\pi ft)$

D. $\exp(-j2\pi f/t)$

Option **C**

f $h(t) = \dfrac{y(t)}{x(t)}$ then $y(t) = x(t) \circledast h(t)$ consider $x(t) = e^{j2\pi ft}$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{j2\pi f(t-\tau)}\, d\tau$$

$$y(t) = e^{j2\pi ft} H(f) \Rightarrow H(f) = \frac{y(t)}{e^{j2\pi ft}}.$$

2.The analog signal m(t) is given below m(t) = 4 cos 100 πt + 8 sin 200 πt + cos 300 πt, the Nyquist sampling rate will be

A. 1/100

B. 1/200

C. 1/300

D. 1/600

Answer: Option **C**

Explanation: m (t) = 4 cos 100 πt + 8 sin 200 πt + cos 300 πt

Nyquist sampling freq $f_s \leq 2f_m$ where f_m is highest frequency component in given signal and highest f_m in 3rd part

$2\pi f_m t = 300\ \pi t$

$f_m = 150$ Hz

f_s = 2 x 150 ⇒ 300 Hz

$$\text{Sampling rate} = \frac{1}{f_s} \Rightarrow \frac{1}{300}\ \text{sec}$$

3 .A function having frequency f is to be sampled. The sampling time T should be

A. $T = \frac{1}{2f}$

B. $T > \frac{1}{2f}$

C. $T < \frac{1}{2f}$

D. $T \geq \frac{1}{2f}$

Answer: Option **C**

Explanation: Sampling frequency must be more than 2f. Therefore $T < \frac{1}{2f}$

4 . Inverse Fourier transform of sgn (ω) is

A. $\frac{j}{\pi t}$

B. 1

C. U(t)

D. $\frac{2}{jt}$

Answer: Option **A**

Explanation: By duality Property.

4. A signal m(t) is multiplied by a sinusoidal waveform of frequency f_c such that v(t)=m(t) cos $2\pi f_c t$. If Fourier transform of m(t) is M(f), Fourier transform of v(t) will be

A. 0.5 M(f + f_c)

B. 0.5 M(f - f_c)

C. 0.5 M(f + f_c) + 0.5 M(f - f_c)

D. $0.5\ M(f - f_c) + 0.5\ M(f - f_c)$

Answer: Option **C**

Explanation: It is a modulation process. The resultant has $(f + f_c)$ and $(f - f_c)$ terms

5. The analog signal given below is sampled by 600 samples per second for m(t) = 3 sin 500t + 2 sin 700 t then folding frequency is

A. 500 Hz

B. 700 Hz

C. 300 Hz

D. 1400 Hz

Answer: Option **C**

Explanation: $\text{Folding frequency } (f_{max}) = \frac{f_{sampling}}{2}$

$\Rightarrow \frac{600}{2} \Rightarrow 300 \text{ Hz}$.

6. The sampling of a function f(l) = sin $2 f_0 t$ starts from a zero crossing. The signal can be detected if sampling time T is

A. $T = \frac{1}{2f_0}$

B. $T > \frac{1}{2f_0}$

C. $T < \frac{1}{2f0}$

D. $T \leq \frac{1}{2f_0}$

Answer: Option **C**

Explanation: Because $f_s \leq 2f_0$, $Ts \leq \frac{1}{2f_0}$.

7. Energy density spectrum of $x[n] = a^n \cup[n]$ for $-1 < a < +1$ is

A. $\left(\frac{1}{1-ae^{-j\omega}}\right)^2$

B. $\left(\frac{1}{1-a}\right)^2$

C. $\frac{1}{1-a}$

D. $\frac{1}{1+a}$

Answer: Option **C**

Explanation: E.S.D. = $|G(f)|^2$

and $G(f) = \frac{1}{1-ae^{-j\omega}} \Rightarrow |G(f)| = \frac{1}{1-a}$

E.S.D. $= \left|\frac{1}{1-a}\right|$

8. The F.T. of a conjugate symmetric function is always

A. Imaginary

B. Real

C. conjugate Unsymmetric

D. conjugate symmetric

Answer: Option **B**

Explanation: F.T. of conjugate symmetric function is always real.

9. Energy density spectrum of a gate $G_T(t)$ function is

A. $A^2T^2\, S_a{}^2\left(\frac{\omega T}{2}\right)$

B. $AT\, S_a\left(\frac{\omega T}{2}\right)$

C. $A^2T^2\, S_a\left(\frac{\omega T}{2}\right)$

D. $AT\, S_a{}^2\left(\frac{\omega T}{2}\right)$

Answer: Option **A**

Explanation:

$\text{F.T. of } [G_T(t)] \Rightarrow G(f) = AT\ sa\ \frac{\omega T}{2}$ and E.S.D. of g(t) = $|G(f)|^2$

So E.S.D. of $G_T(t) = |G(f)|^2 = A^2T^2\, sa^2 \frac{\omega T}{2}$

10. The signal define by the equations u(t - a) = 0 for t < a and u(t - a) = 1 for t ≥ a is

A. a unit step function

B. a shifted unit step function orginating at t = a

C. a pulse function

D. none of the above

Answer: Option **B**

Explanation: u(t) is a unit step function u(t - a) is a unit step function shifted in time by a.

Fill in the Blanks

1. If F(jω) is the Fourier transform of f(t), then.................. **£ f(- t) = F* (jω)**
2. If f(t) = 1, F(jω) then.............. **2□δ (□)**
3. For a Gaussian process, auto-correlation sequence also implies that…statistical dependence
4. The energy associated with a function f(t) is $E = \int_{-\infty}^{\infty} f^2(t)\, dt$. In terms of Fourier transform, E is................ $\frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$
5. If f(t) is an odd function, F(jω) .... $2j\int_{0}^{\infty} f(t)\sin(\omega t)dt$
6. If f(t) ↔ F(jω), $\frac{d^n}{dt^n} f(t)$ ↔.................... **$(j\omega)^n F(j\omega)$**

7. Let $x(t) \leftrightarrow X(j\omega)$ be F.T pair. The Fourier transform of the signal $x(5t - 3)$ in terms of $X(j\omega)$ is given as……………… $\frac{1}{5}e^{\frac{-j3\omega}{5}}X\left(\frac{j\omega}{5}\right)$
8. If f(t) and F(jω) form a transform pair, then as per symmetry in Fourier transforms

F(jt) ↔ 2?f (- ω)

9. If $f_1(t)$<< $F_1(j\omega)$ and $f_2(t) \leftrightarrow F_2(j\omega)$, then $[a_1 f_1(t) + a_2f_2(t)] \leftrightarrow$……………….. $\mathbf{a_1F_1(j\omega) + a_2F_2(j\omega)}$
10. The inverse Fourier transform of δ(t) is…………….. **1**

Objective type Questions:

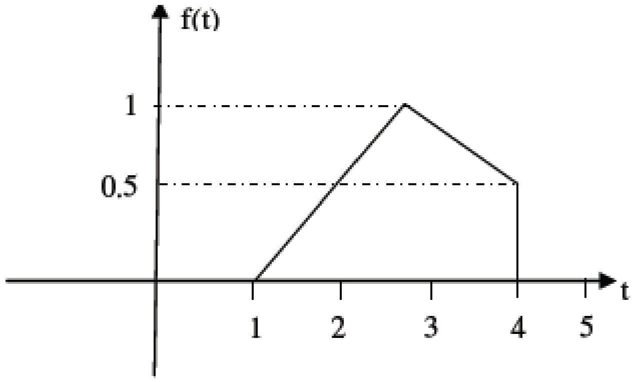
1. Define Fourier transform pair.
2. Write short notes on dirichlets conditions for Fourier transform.
3. State convolution property in relation to fourier transform.
4. State parseval's relation for continuous time fourier transform.
5. What is meant by sampling.
6. State Sampling theorem.
7. What is meant by aliasing?
8. What are the effects aliasing?
9. Define Nyquist rate and Nyquist interval.
10. Define sampling of band pass signals.

Analytical Questions:

1. How aperiodic signals can be represented by fourier transform.
2. Why CT signals are represented by samples.
3. How the aliasing process is eliminated.
4. Can Sampling applicable to High Frequency Signals.
5. Need of Transforms.

Essay Answer Questions:

1. State and prove differentiation property of Fourier transforms.
2. Find the fourier transform of following waveform using the property of Fourier transform.



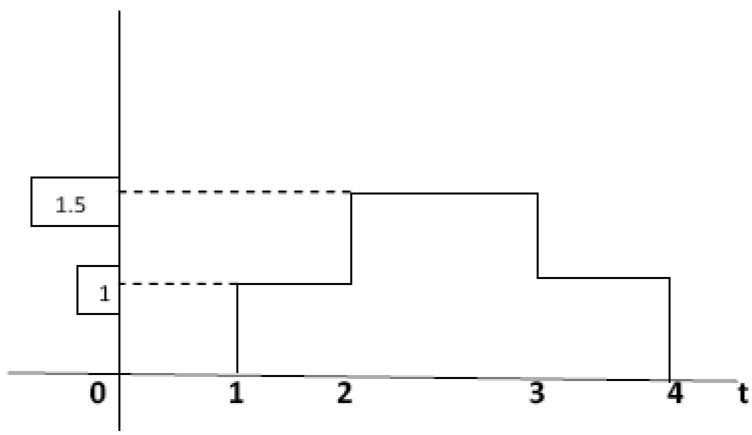
i.

3. State and Prove sampling theorem for low pass signals. Also, explain the recovery of original signal from its sampled signal. Draw neat diagrams wherever necessary.
4. Obtain the Nyquist rate of the signal, x(t) = cos 2000p t +10sin 10000p t + 20 cos 5000p t
5. Define Nyquist rate. Compare the merits and demerits of performing sampling using impulse, natural and flat top sampling techniques.
6. Using the properties of Fourier transforms, compute the FT of the following

   i) $x(t) = \sin(2\pi t)\, e^{-t} u(t)$ ii) $x(t) = t\, e^{-3|t-1|}$
7. State all the properties of Fourier Transforms.

Problems:

1. Evaluate the Fourier transform of the signal shown bellow.



2. Determine the Fourier transform of a signal x(n)=cos (w0t) u(t)

3. Consider the response of an LTI system with impulse response h (t) =e-atu(t) for a>0 to the input signal x(t)= e-btu(t) for b>0. Determine the output of the system using convolution property.

4. Find the Fourier Transform of the following signals? Which of these signals have Fourier Transform that converge? Which of these signals have Fourier Transform that are real? Imaginary?

$$x(t) = \cos(1000t)$$
$$x(t) = 3\,\delta(t)$$
$$x(t) = 13\cos(100t) - 7\sin(500t)$$

11. Using convolution theorem, find the inverse Fourier transform of X(W)= 1/(a+jw)2

Skill Building Exercises/Assignments

1. Using convolution theorem, find the inverse Fourier transform of X(W)= 1/(a+jw)2
2. Determine the inverse Fourier transform of X(jw)=2πδ(w)+ πδ(w-4π)+ πδ(w+4π).
3. Determine the frequency spectrum of the positive time function e-t sin (100t) ; t >o
4. Find the Fourier Transform of the Traingular Pulse
5. Consider a causal LTI system with frequency response H(jw)=1/(jw+3). For a particular input x(t) this system is observed to produce the output y(t)=e-3tu(t)- e-4tu(t).Determine x(t).
6. Find the Hilbert transform of x(t)=sinw0t.

Previous Paper Questions:

1. Find the Fourier transform of the signum function and plot its amplitude and phase spectra.
2. State and prove the following properties of Fourier Transform
   i) Time shifting ii) Convolution in time domain
3. Explain how Fourier transform is developed from Fourier series.
4. Obtain the Fourier transform of the following functions.
   i) Impulse function ii) DC signal iii) Unit step function.
5. Explain the characteristics of an ideal LPF. All ideal filters are physically not realizable: justify.
6. State and prove sampling theorem for band limited signals using graphical approach.
7. What is aliasing? Explain its effect on sampling.
8. Find the Fourier transform of the following functions.
   i) A single symmetrical triangular pulse. ii) A single symmetrical gate pulse.
9. State the conditions for the existence of Fourier transform of a signal.
10. What is an ideal filter and Find impulse response of an ideal Low Pass Filter? Obtain the relationship between the bandwidth and rise time of ideal low pass filte**r.**

4.2.7. Gate Questions:

12. The Fourier Series of an odd periodic function, contains only

a. Odd harmonics

b. even harmonics

c. cosine terms

d. sine terms GATE 1994

13. 2. The RMS value of a rectangular wave of period T, having a value of +V for a duration, T1(<T) and –V for the duration, T-T1 = T2 equals

a. V

b. (T1-T2)/T V

c. $V/\sqrt{2}$

d. T1 /T V GATE 1995

14. The Trigonometric Fourier Series of a periodic time function can have only

a. cosine terms

b. sine terms

c. cosine & sine terms

d. DC& cosine terms GATE 1998

15. Which of the following cannot be the Fourier Series expansion of a periodic signal?

a. $x(t) = 2 \cos t + 3 \cos 3t$

b. $x(t) = 2 \cos \pi t + 7 \cos t$

c. $x(t) = \cos t + 0.5$

d. $x(t) = 2 \cos 1.5\pi t + \sin 3.5 \pi t$ GATE 2002

16. Choose the function f(t); $-\infty < t < +\infty$, for which a Fourier Series cannot be defined

a. 3 sin (25t)

b. 4 cos(20t+3) +3 sin (10t)

c. exp(-|t|)sin (25t)

d. 1 GATE 2005

17. The Trigonometric Fourier Series of an even function of time does not have

a. d.c. terms

b. cosine terms

. sine terms

d. odd harmonic terms GATE 1996,2011

18. Trigonometric Fourier Series of a real periodic function has only

a. cosine terms if it is even

b. sine terms if it is even

c. cosine terms if it is odd

d. sine terms if it is odd Which of the above statements are correct?

a. P and S b. P and R c. Q and S d. Q and R GATE 2009

19. A function is given by f(t) = sin2t + cos 2t. which of the following is true? GATE 2009

1.5. f has frequency components at 0 and 1/2π Hz

1.6. **f has frequency components at 0 and 1/π Hz**

1.7. c. f has frequency components at 1/2π&1/π Hz

1.8. f has frequency components at 0,1/2π &1/π Hz

20. A half – wave rectified sinusoidal waveform has a peak voltage of 10 V. Its average value and the peak value of the fundamental component are respectively given by:

a. 20/ π V, 10/√2 V

b. 10/π V, 10/√2 V

c. 10/ π V, 5 V

. 20/ π V, 5 V GATE 1987

21. Which of the following signals is/are periodic? GATE 1992

a. s(t) = cos 2t + cos 3t + cos 5t

b. s(t) = exp(j8πt)

c. s(t) = exp(-7t)sin 10πt

d. s(t) = cos 2t cos 4t

22. The PSD and the power of a signal g(t), are respectively, Sg(w), Pg. The PSD and the power of the signal a g(t) are, respectively,

a. a2Sg(w), a2 Pg

b. a2Sg(w), a Pg

c. aSg(w), a2 Pg

d. aSg(w), a Pg GATE 2001

Unit 3

Linear system, impulse response, Response of a linear system, Linear time invariant (LTI) system, Linear time variant (LTV) system, Transfer function of a LTI system, Filter characteristics of linear systems, Distortion less transmission through a system, Signal bandwidth, system bandwidth, Ideal LPF, HPF and BPF characteristics, Causality and Poly-Wiener criterion for physical realization, Relationship between bandwidth and rise time.

Signals & Systems Concepts

Systems, signals, mathematical models. Continuous-time and discrete-time signals. Energy and power signals. Linear systems. Examples for use throughout the course

Specific objectives:

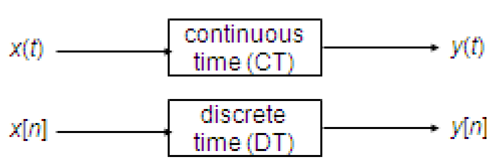
- Introduction to systems
- Continuous and discrete time systems
- Properties of a system
- Linear (time invariant) LTI systems
- Linear Systems

A system takes a signal as an input and transforms it into another signal

Linear systems play a crucial role in most areas of science

- Closed form solutions often exist
- Theoretical analysis is considerably simplified
- Non-linear systems can often be regarded as linear, for small perturbations, so-called linearization

We are going to be considering Linear, Time Invariant systems (LTI) and consider their properties



Examples of Simple Systems

To get some idea of typical systems (and their properties), consider the electrical circuit example:

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

which is a first order, CT differential equation.

Examples of first order, DT difference equations:

$$y[n] = x[n] + 1.01 y[n-1]$$

where y is the monthly bank balance, and x is monthly net deposit

$$v[n] - \frac{RC}{RC+k} v[n-1] = \frac{k}{RC+k} f[n]$$

which represents a discretised version of the electrical circuit

Example of second order system includes:

$$a\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + cy(t) = x(t)$$

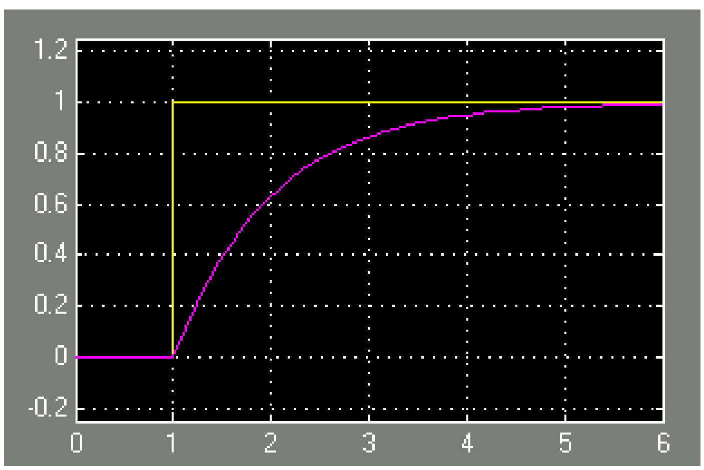
System described by order and parameters (a, b, c)

First Order Step Responses:

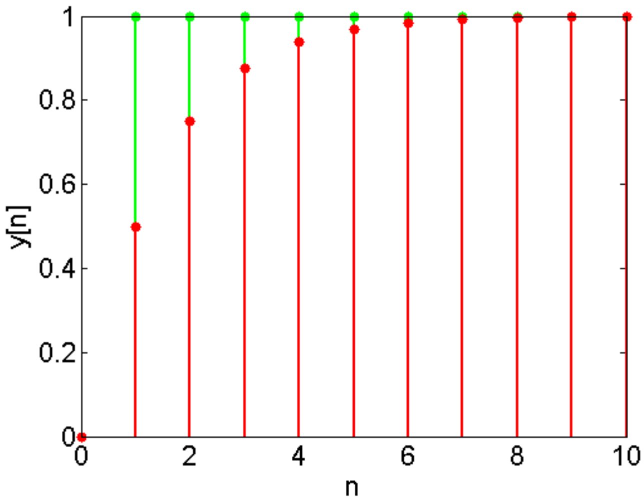
The dynamics of the output signal are determined by the dynamics of the system, if the input signal has no dynamics

Consider when the input signal is a step at t, n = 1, y(0) = 0

$$\frac{dy(t)}{dt} + ay(t) = u(t-1)$$



$$y[n](1+ak) - y[n-1] = ku[n-1]$$



System Linearity

Specifically, a linear system must satisfy the two properties:

Additive: the response to $x_1(t)+x_2(t)$ is $y_1(t) + y_2(t)$

Combined: $ax_1(t)+bx_2(t) \rightarrow ay_1(t) + by_2(t)$

Scaling: the response to $ax_1(t)$ is $ay_1(t)$ where $a \in C$

E.g. Linear $y(t) = 3*x(t)$ why?

Non-linear $y(t) = 3*x(t)+2$, $y(t) = 3*x^2(t)$ why?

(equivalent definition for DT systems)

Bias and Zero Initial Conditions

Intuitively, a system such as:

$y(t) = 3*x(t)+2$

is regarded as being linear. However, it does not satisfy the scaling condition.

There are several (similar) ways to transform it to an equivalent linear system

Perturbations around operating value x^*, y^*

$$\varepsilon_x(t) = x(t) - x^*, \quad \varepsilon_y(t) = y(t) - y^*$$
$$\varepsilon_y(t) = 3*\varepsilon_x(t)$$

Linear System Derivative

$$\partial y(t) = 3\partial x(t)$$

Locally, these ideas can also be used to linearise a non-linear system in a small range

Linearity and Superposition

Suppose an input signal x[n] is made of a linear sum of other (basis/simpler) signals $x_k[n]$:

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + a_3 x_3[n] + \cdots$$

then the (linear) system response is:

$$y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n] + \cdots$$

The basic idea is that if we understand how simple signals get affected by the system, we can work out how complex signals are affected, by expanding them as a linear sum. This is known as the superposition property which is true for linear systems in both CT & DT.

Definition of Time Invariance

A system is time invariant if its behaviour and characteristics are fixed over time

We would expect to get the same results from an input-output experiment, if the same input signal was fed in at a different time

E.g. The following CT system is time-invariant

$$y(t) = \sin(x(t))$$

because it is invariant to a time shift, i.e. $x_2(t) = x_1(t-t_0)$

$$y_2(t) = \sin(x_2(t)) = \sin(x_1(t-t_0)) = y_1(x_1(t-t_0))$$

E.g. The following DT system is time-varying

$$y[n] = nx[n]$$

Because the system parameter that multiplies the input signal is time varying, this can be verified by substitution

$$x_1[n] = \delta[n] \Rightarrow y_1[n] = 0$$
$$x_2[n] = \delta[n-1] \Rightarrow y_2[n] = \delta[n-1]$$

System with and without Memory

A system is said to be memoryless if its output for each value of the independent variable at a given time is dependent on the output at only that same time (no system dynamics)

$$y[n] = (2x[n] - x^2[n])^2$$

e.g. a resistor is a memoryless CT system where x(t) is current and y(t) is the voltage

A DT system with memory is an accumulator (integrator)

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

and a delay

$$y[n] = x[n-1]$$

Roughly speaking, a memory corresponds to a mechanism in the system that retains information about input values other than the current time.

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{n-1} x[k] + x[n] \\ &= y[n-1] + x[n] \end{aligned}$$

System Causality

A system is causal if the output at any time depends on values of the output at only the present and past times. Referred to as non-anticipative, as the system output does not anticipate future values of the input

If two input signals are the same up to some point t_0/n_0, then the outputs from a causal system must be the same up to then.

E.g. The accumulator system is causal:

because y[n] only depends on x[n], x[n-1], …

E.g. The averaging/filtering system is non-causal

because y[n] depends on x[n+1], x[n+2], …

Most physical systems are causal

System Stability

Informally, a stable system is one in which small input signals lead to responses that do not diverge

If an input signal is bounded, then the output signal must also be bounded, if the system is stable

$$\forall x : |x| < U \rightarrow |y| < V$$

To show a system is stable we have to do it for all input signals. To show instability, we just have to find one counterexample

E.g. Consider the DT system of the bank account

$$y[n] = x[n] + 1.01 y[n-1]$$

when x[n] = d[n], y[0] = 0

This grows without bound, due to 1.01 multiplier. This system is unstable.

E.g. Consider the CT electrical circuit, is stable if RC>0, because it dissipates energy

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} v_s(t)$$

Invertible and Inverse Systems

A system is said to be invertible if distinct inputs lead to distinct outputs (similar to matrix invertibility). If a system is invertible, an inverse system exists which, when cascaded with the original system, yields an output equal to the input of the first signal

E.g. the CT system is invertible: y(t) = 2x(t) . because w(t) = 0.5*y(t) recovers the original signal x(t)

E.g. the CT system is not-invertible y(t) = x^2(t). because distinct input signals lead to the same output signal

Widely used as a design principle:

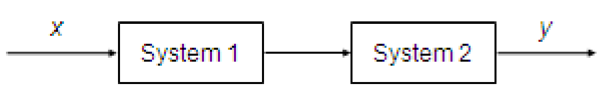
- Encryption, decryption
- System control, where the reference signal is input

System Structures

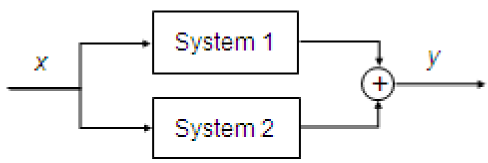
Systems are generally composed of components (sub-systems).

We can use our understanding of the components and their interconnection to understand the operation and behaviour of the overall system

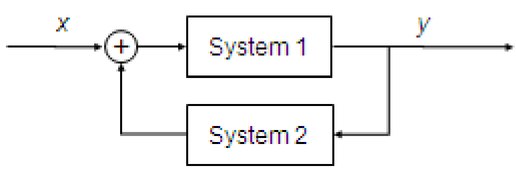
Series/cascade



Parallel

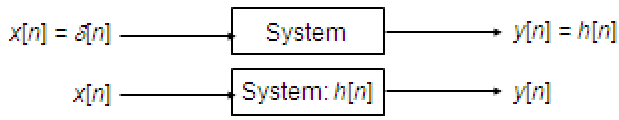


Feedback



Linear Systems and Convolution

Convolution is an operator that takes an input signal and returns an output signal, based on knowledge about the system's unit impulse response h[n].



The basic idea behind convolution is to use the system's response to a simple input signal to calculate the response to more complex signals

$$x[n] = \sum_k a_k x_k[n] = a_1 x_1[n] + a_2 x_2[n] + a_3 x_3[n] + \cdots$$

$$y[n] = \sum_k a_k y_k[n] = a_1 y_1[n] + a_2 y_2[n] + a_3 y_3[n] + \cdots$$

Discrete Impulses & Time Shifts

Basic idea: use a (infinite) set of of discrete time impulses to represent any signal.

Consider any discrete input signal x[n]. This can be written as the linear sum of a set of unit impulse signals:

$$x[-1]\delta[n+1] = \begin{cases} x[-1] & n = -1 \\ 0 & n \neq -1 \end{cases}$$

$$x[0]\delta[n] = \begin{cases} x[0] & n = 0 \\ 0 & n \neq 0 \end{cases}$$

$$x[1]\delta[n-1] = \begin{cases} x[1] & n = 1 \\ 0 & n \neq 1 \end{cases}$$

Therefore, the signal can be expressed as:

$$x[n] = \cdots + x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \cdots$$

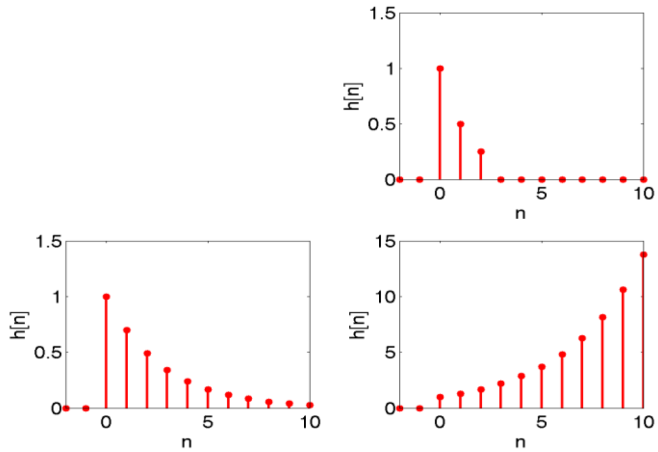
In general, any discrete signal can be represented as:

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

Discrete, Unit Impulse System Response

A very important way to analyse a system is to study the output signal when a unit impulse signal is used as an input. The output signal can be used to infer properties about the system's structure and its parameters q.

Types of Unit Impulse Response



Linear, Time Varying Systems

If the system is time varying, let $h_k[n]$ denote the response to the impulse signal d[n-k] (because it is time varying, the impulse responses at different times will change). Then from the superposition property of linear systems, the system's response to a more general input signal x[n] can be written as:

Input signal $$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

System output signal is given by the convolution sum $$y[n] = \sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

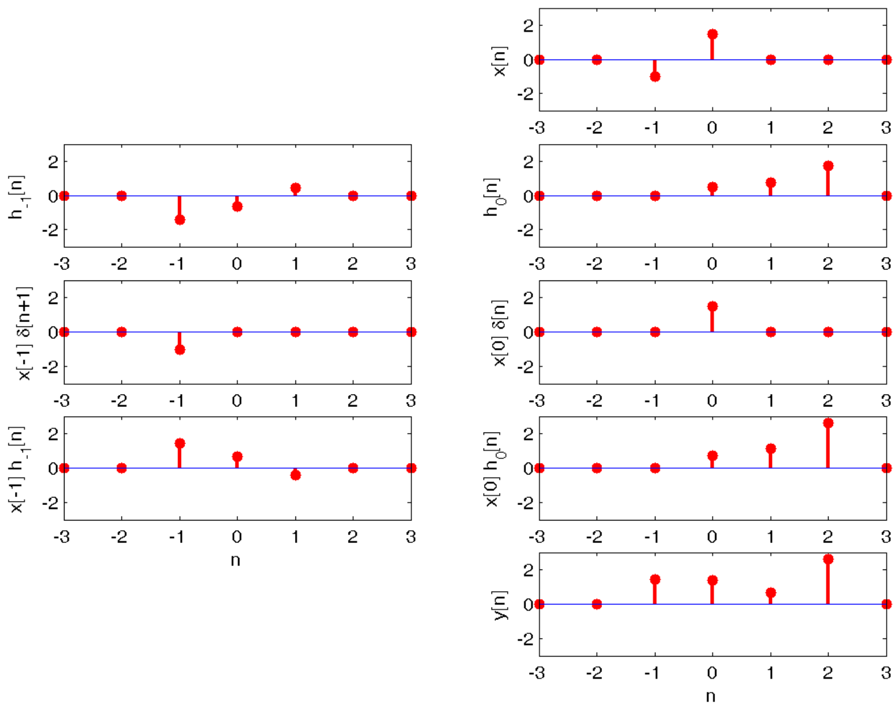
i.e. it is the scaled sum of impulse responses

Example: Time Varying Convolution

x[n] = [0 0 –1 1.5 0 0 0]

h_{-1}[n] = [0 0 –1.5 –0.7 .4 0 0]

h_0[n] = [0 0 0 0.5 0.8 1.7 0]



Linear Time Invariant Systems

When system is linear, time invariant, the unit impulse responses are all time-shifted versions of each other:

$$h_k[n] = h_0[n-k]$$

It is usual to drop the 0 subscript and simply define the unit impulse response h[n] as:

$$h[n] = h_0[n]$$

In this case, the convolution sum for LTI systems is:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

It is called the convolution sum (or superposition sum) because it involves the convolution of two signals x[n] and h[n], and is sometimes written as:

$$y[n] = x[n] * h[n]$$

System Identification and Prediction

Note that the system's response to an arbitrary input signal is completely determined by its response to the unit impulse. Therefore, if we need to identify a particular LTI system, we can apply a unit impulse signal and measure the system's response. That data can then be used to predict the system's response to any input signal

Note that describing an LTI system using h[n], is equivalent to a description using a difference equation. There is a direct mapping between h[n] and the parameters/order of a difference equation such as:

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2]$$

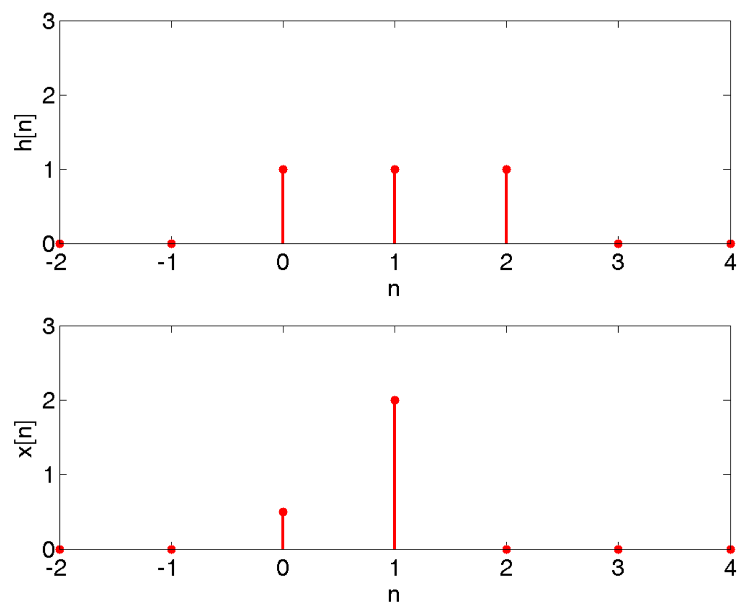
LTI Convolution

Consider a LTI system with the following unit impulse response:

h[n] = [0 0 1 1 1 0 0]

For the input sequence:

x[n] = [0 0 0.5 2 0 0 0]



The result is:

y[n] = ... + x[0]h[n] + x[1]h[n-1] + ...

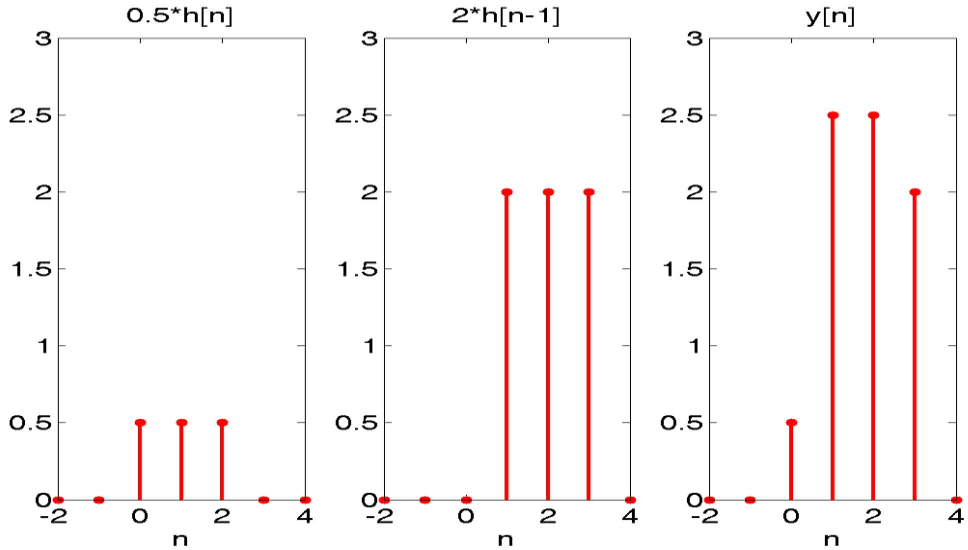
= 0 +

0.5*[0 0 1 1 1 0 0] +

2.0*[0 0 0 1 1 1 0] +

0

= [0 0 0.5 2.5 2.5 2 0]



Consider the problem described for example 1

Sketch x[k] and h[n-k] for any particular value of n, then multiply the two signals and sum over all values of k.

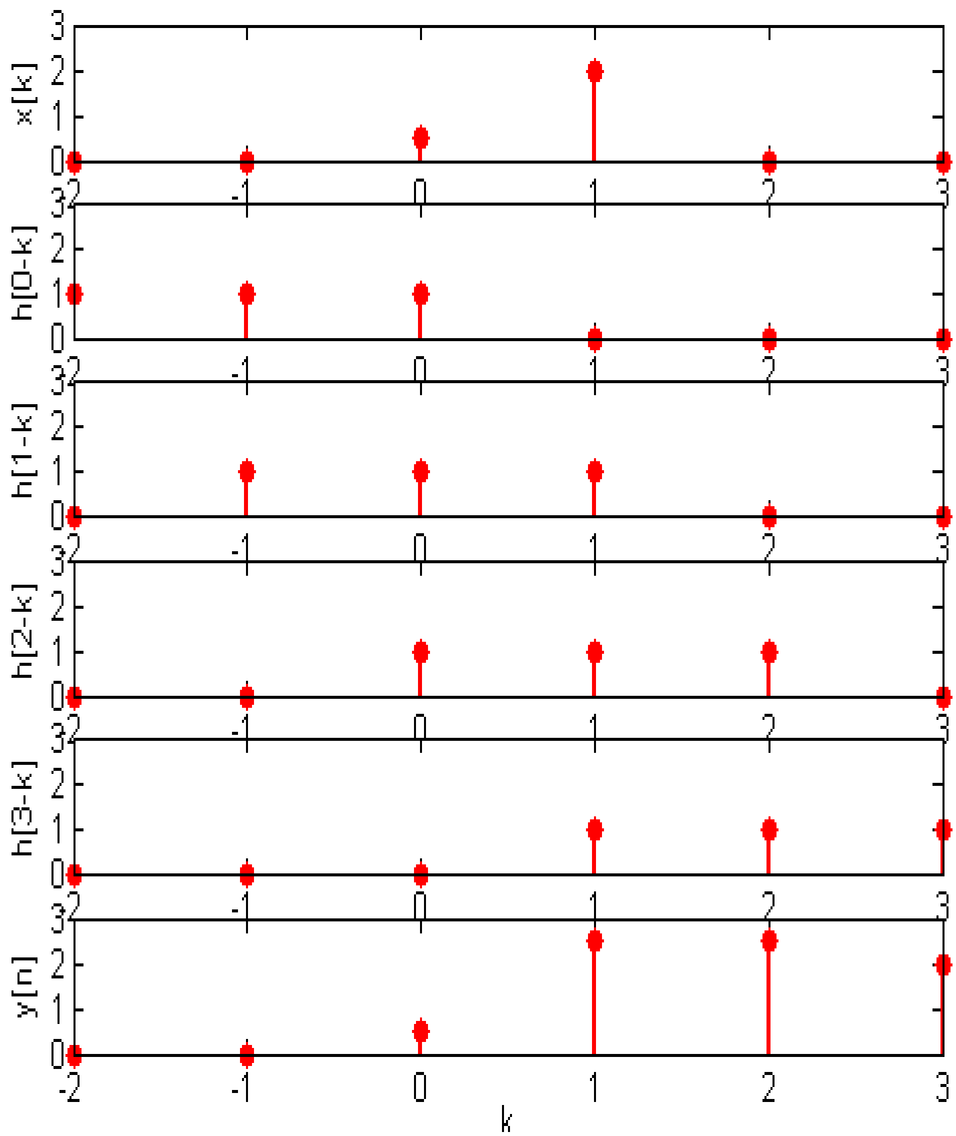
For n<0, we see that x[k]h[n-k] = 0 for all k, since the non-zero values of the two signals do not overlap.

$y[0] = S_k x[k]h[0-k] = 0.5$

$y[1] = S_k x[k]h[1-k] = 0.5+2$

$y[2] = S_k x[k]h[2-k] = 0.5+2$

$y[3] = S_k x[k]h[3-k] = 2$



Example 3: LTI Convolution

Consider a LTI system that has a step response h[n] = u[n] to the unit impulse input signal

What is the response when an input signal of the form

$x[n] = a^n u[n]$

where 0<a<1, is applied?

$$y[n] = \sum_{k=0}^{n} \alpha^k$$
$$= \frac{1-\alpha^{n+1}}{1-\alpha}$$

For n≥0:

Therefore,

$$y[n] = \left(\frac{1-\alpha^{n+1}}{1-\alpha}\right)u[n]$$

Signal "Staircase" Approximation

As previously shown, any continuous signal can be approximated by a linear combination of thin, delayed pulses:

$$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & 0 \le \mathrm{t} < \Delta \\ 0 & \text{otherwise} \end{cases}$$

Note that this pulse (rectangle) has a unit integral. Then we have:

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

Only one pulse is non-zero for any value of t. Then as D→0

$$x(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

When D→0, the summation approaches an integral

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

This is known as the sifting property of the continuous-time impulse and there are an infinite number of such impulses d(t-t)

Alternative Derivation of Sifting Property

The unit impulse function, d(t), could have been used to directly derive the sifting function.

$$\delta(t-\tau) = 0 \qquad t \neq \tau$$

$$\int_{-\infty}^{\infty} \delta(t-\tau)d\tau = 1$$

Therefore:

$$\begin{aligned} x(\tau)\delta(t-\tau) &= 0 \quad t \neq \tau \\ \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau &= \int_{-\infty}^{\infty} x(t)\delta(t-\tau)d\tau \\ &= x(t)\int_{-\infty}^{\infty} \delta(t-\tau)d\tau \\ &= x(t) \end{aligned}$$

The previous derivation strongly emphasises the close relationship between the structure for both discrete and continuous-time signals.

Continuous Time Convolution

Given that the input signal can be approximated by a sum of scaled, shifted version of the pulse signal, $d_D(t-kD)$

$$\hat{x}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

The linear system's output signal y is the superposition of the responses, $h_{kD}(t)$, which is the system response to $d_D(t-kD)$.

$$\hat{y}(t) = \sum_{k=-\infty}^{\infty} x(k\Delta)\hat{h}_{k\Delta}(t)\Delta$$

What remains is to consider as $D\rightarrow 0$. In this

From the discrete-time convolution: case:

$$y(t) = \lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\hat{h}_{k\Delta}(t)\Delta$$

$$= \int_{-\infty}^{\infty} x(\tau)h_\tau(t)d\tau$$

Linear Time Invariant Convolution

For a linear, time invariant system, all the impulse responses are simply time shifted versions:

Therefore, convolution for an LTI system is defined by:

$$h_\tau(t) = h(t-\tau)$$

This is known as the convolution integral or the superposition integral

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Algebraically, it can be written as:

$$y(t) = x(t) * h(t)$$

To evaluate the integral for a specific value of t, obtain the signal h(t-t) and multiply it with x(t) and the value y(t) is obtained by integrating over t from $-\infty$ to ∞.

Demonstrated in the following examples

CT Convolution

Let x(t) be the input to a LTI system with unit impulse response h(t):

$$x(t) = e^{-at}u(t) \qquad a > 0$$
$$h(t) = u(t)$$

For t>0:

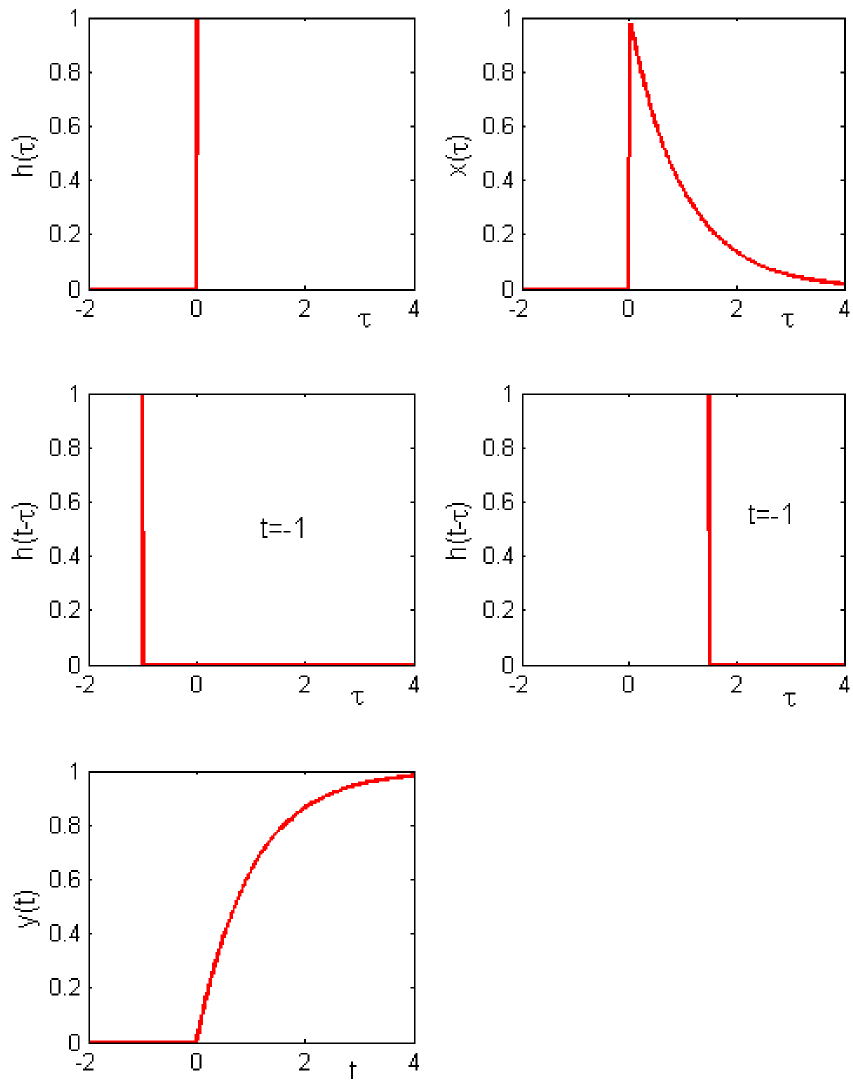
$$x(\tau)h(t-\tau) = \begin{cases} e^{-a\tau} & 0 < \tau < t \\ 0 & \text{otherwise} \end{cases}$$

We can compute y(t) for t>0:

$$y(t) = \int_0^t e^{-a\tau} d\tau = -\tfrac{1}{a} e^{-a\tau}\Big|_0^t$$
$$= \tfrac{1}{a}\left(1 - e^{-at}\right)$$

So for all t:

$$y(t) = \tfrac{1}{a}\left(1 - e^{-at}\right)u(t)$$



Multiple Choice Questions

1

If $y(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{X(\tau)}{t - \tau} d\tau$, then F[$y(t)$] is

A. $-j$ sgn (f) x X(f)

B. $j2\pi f$ x $e^{j\pi tf}$ X(f)

C. $\frac{1}{\pi}$ x sinc (ft) x X(f)

D. $\frac{\sin(\pi + \tau)}{\pi f \tau}$ x X (f)

Option **A**

2

Which of the following is/are not a property/properties power spectral density function Sx(ω)?

A. Sx(ω) is real function of ω

B. Sx(ω) is a even function of ω

C. Sx(ω) is non-positive function of ωSx(ω) ≤ 0 for all ω

D. All of the above

Option **C**

3

A casual LTI system is described by the difference equation
$2y[n] = \alpha y[n - 2] - 2x[n] + \beta x[n - 1]$
The system is stable only if

A. $|\alpha| = 2$ $|\beta| < 2$

B. $|\alpha| > 2$ $|\beta| > 2$

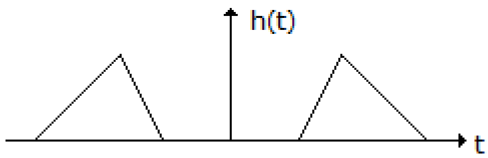
C. $|\alpha| < 2$ any value of β

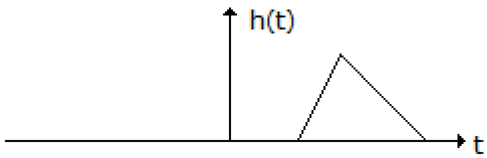
D. $|\beta| < 2$, any value of α

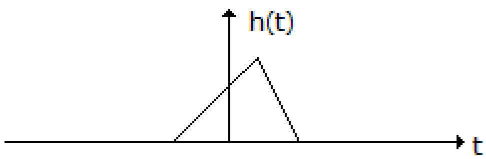
Option **C**

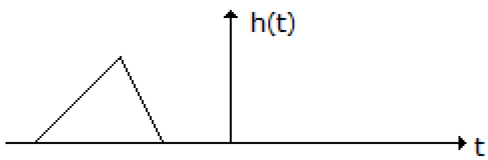
4

Which of the following can be impulse response of a casual system?

A. 

B. 

C. 

D. 

Option **B**

5

An excitation is applied to a system at t = T and the response in zero for $-\infty < t < T$. This system is

A. non casual

B. Stable

C. Casual

D. Unstable

Option **C**

6

The signumm function written as [$sgn(t)$] is defined as

A. $sgn(t)$ = - 1 for $t < 0$, = 0 for $t = 0$ and = 1 for $t > 0$

B. $sgn(t)$ = 1 for $t < 0$, = 0 for $t = 0$ and = - 1 for $t > 0$

C. $sgn(t)$ = 0 for $t < 0$, = 1 for $t = 0$ and = 0 for $t > 0$

D. $sgn(t)$ = 0 for $t < 0$, = 1 for t^3 0

Option **A**

7

The units of F($j\omega$) are volt-seconds.

A. True

B. False

Option **A**

8

A system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin 5/6\ \pi n)\ x(n)$ The system is

A. linear, stable and invertible

B. non-linear, stable and non-invertible

C. linear stable, non invertible

D. linear, unstable, invertible

Option **C**

9

The output $y(t)$ of a linear time invariant system is related to its input $x(t)$ by the following equation $y(t) = 0.5x(t - t_d + 1) + x(t - t_d) + 0.5\ x(t - t_d + 7)$. The filter transfer function $H(\omega)$ of such a system is given by

A. $(1 + \cos \omega t)e^{-j\omega t}\ d$

B. $(1 + 0.5 \cos \omega t)e^{-j\omega t}\ d$

C. $(1 + \cos \omega t)e^{j\omega t}\ d$

D. $(1 - 0.5 \cos \omega t)e^{-j\omega t}\ d$

Option **A**

10

Final value theroem is for sequence $x[n]$ is

A. $\lim_{n \to \infty} x[n]$

B. $\lim_{z \to 1} (z-1)\,x(z)$

C. a and b

D. $X(z)_{\text{at } z = \infty}$

Option **C**

Fill in the Blanks

1 The units of F($j\omega$) are……………………..

Sol volt-seconds

2 The signumm function written as [*sgn(t)*] is defined as……………………..

Sol *sgn(t)* = - 1 for $t < 0$, = 0 for $t = 0$ and = 1 for $t > 0$

3 system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin 5/6\ \pi n)\ x(n)$…………..

Sol linear

4 system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin 5/6\ \pi n)\ x(n)$…………..

Sol stable

5 system with input $x[n]$ and output $y[n]$ is given as $y[n] = (\sin 5/6\ \pi n)\ x(n)$…………..

Sol non invertible

6 Examples of first order, DT difference equations is………………..

Sol $y[n] = x[n] + 1.01 y[n-1]$

7 Example of second order system includes is…………..

Sol $$a\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + cy(t) = x(t)$$

8 An excitation is applied to a system at t = T and the response in zero for $-\infty < t < T$. This system is………

Sol causal

9 Which of the following signals are periodic………..

Sol cos 2t + cos 3t + cos 5t

10 any continuous signal can be approximated by a linear combination of thin, delayed pulses

Sol $$\delta_\Delta(t) = \begin{cases} \frac{1}{\Delta} & 0 \le t < \Delta \\ 0 & \text{otherwise} \end{cases}$$

Objective type questions

1. State Convolution property.
2. Define a causal system.
3. What is meant by linear system?
4. Define time invariant system.
5. Define stable system?
6. Define memory and memory less system.
7. Define invertible system.

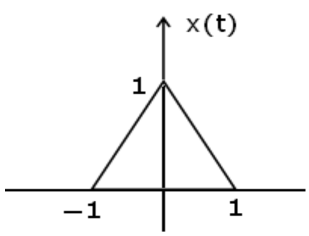
8. What is superposition property?
9. State Convolution property.
10. Define a causal system.
11. What is meant by linear system?
12. Define time invariant system.
13. Define stable system?
14. Define memory and memory less system.
15. Define invertible system.
16. What is superposition property?

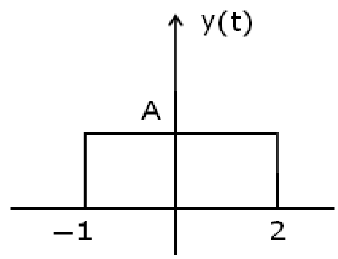
Analytical Questions:

1. Explain the difference between a time invariant system and time variant system? Write some practical cases where you can find such systems.
2. What do you understand by the filter characteristics of a linear system?
3. Illustrate the energy and power signals.

Essay Questions

1. Find the convolution using graphical method of the following two signals





2. A Signal $x(t)=100\cos\left(24\pi\times10^{3}t\right)$ is ideally sampled with sampling period of $50\mu\text{sec}$ and then passed through an ideal LPF with cutoff frequency of 15KHz. What are the frequencies that exist in the output of the filter.

2.Let the input of the system $x(t)=e^{-3t}u(t-2)$ and the impulse response $n(t)=e^{-6t}u(t-3)$. Calculate the output of the system.

3.Assume a signal $x(t)=6\cos10\pi t$ sampled at 7Hz and 14Hz. Illustrate the effect of sampling a signal at both frequency less than and greater than twice the highest frequency. Plot the output of the reconstruction filter.

4. Explain causality and physical reliability of a system and explain poly- wiener criterion.
5. Obtain the relationship between the bandwidth and rise time of ideal High pass filter.
6. Differentiate between linear and non-linear system.
7. Define Linearity and Time-Invariant properties of a system
8. Test whether y(t) $d^2y/dt^2 + y(t) = x(t)$ is linear, causal and time invariant or not.
9. Check whether x(t) = $e^{j(2t+\pi/4)}$ is an energy signal or power signal.

Skill Building Exercises/ Assignments

1. State and prove Parseval's Theorem.
2. Find the convolution of two signals x(n) = {2, 1, 0, -1, 1} and h(n) = {1, -2, -3, 5}and represent them graphically.

3. Explain Energy Density Spectrum of a signal and also explain about energy densities of the input and the response of a system.

4. Obtain the relationship between auto correlation and Energy Density Spectrum of a signal.

Previous Paper Questions:

1. Give the condition for Poly – Wiener criterion.

2. What is an LTI system? Explain its properties. Derive an expression for the transfer function of an LTI system.
3. Obtain conditions for the distortion less transmission through a system.

4. Give the relation between autocorrelation and ESD.

5. Explain the characteristics of an ideal LPF. All ideal filters are physically not realizable: justify.
6. Explain how Impulse Response and Transfer Function of a LTI system are related.
7. Let the system function of a LTI system be 1/jw+2. What is the output of the system for an input (0.8)t u(t).
8. Derive the relation between bandwidth and rise time.
9. Define Impulse response of a system and write the expression for transfer function in terms of input signal and output signal.
10. List the steps involved in linear convolution

11. Derive the relation between PSDs of input and output for an LTI system
12. list the filter characteristics of linear systems

13. Obtain the conditions for the distortion less transmission through a system.

14. Show that the cross correlation of *f(t)* with *_(t – t0)* is equal to *f(t – t0)*. Where *_(t–t0)* is delayed unit impulse function.
15. Show that the auto-correlation function at the origin is equal to the energy of the function.

16. Define stability and causality of an LTI system

17. Find the response of an ideal low pass filter when unit step signal is applied as an input.
18. What are the requirements of a system to allow the distortion less transmission of a

Gate Questions

1 The PSD and the power of a signal g(t), are respectively, Sg(w), Pg. The PSD and the power of the signal a g(t) are, respectively,

a. a2Sg(w), a2 Pg b. a2Sg(w), a Pg c. aSg(w), a2 Pg d. aSg(w), a Pg

2 consider the function f(t) having the L.T. [ ]

F(s) = w0 Re[s]>0 the final value of f(t) would be (s2+ w0 2)

a. 0 b. 1 c. -1≤ f(∞) ≤ 1 d. ∞ Ans: C GATE 2006

3 The input and output of a continuous time system are respectively denoted by $x(t)$ and $y(t)$. Which of the following descriptions is corresponds to a causal system? [GATE-2008]

(a) $y(t) = x(t-2) + x(t+4)$ (b) $y(t) = (t-4)\, x(t+1)$ (c) $y(t) = (t+4)\, x(t-1)$ (d) $y(t) = (t+5)\, x(t+5)$
Answer: c

4 The frequency response $H(w)$ of this system in terms of angular frequency w,is given by, $H(w)=$

(a) $1/(1+j2w)$ (b) $(\sin w)/w$ (c) $1/(2+jw)$ (d) $jw/(2+jw)$

Answer: c